Chapter 6

Proportions and Similarity

What You'll Learn

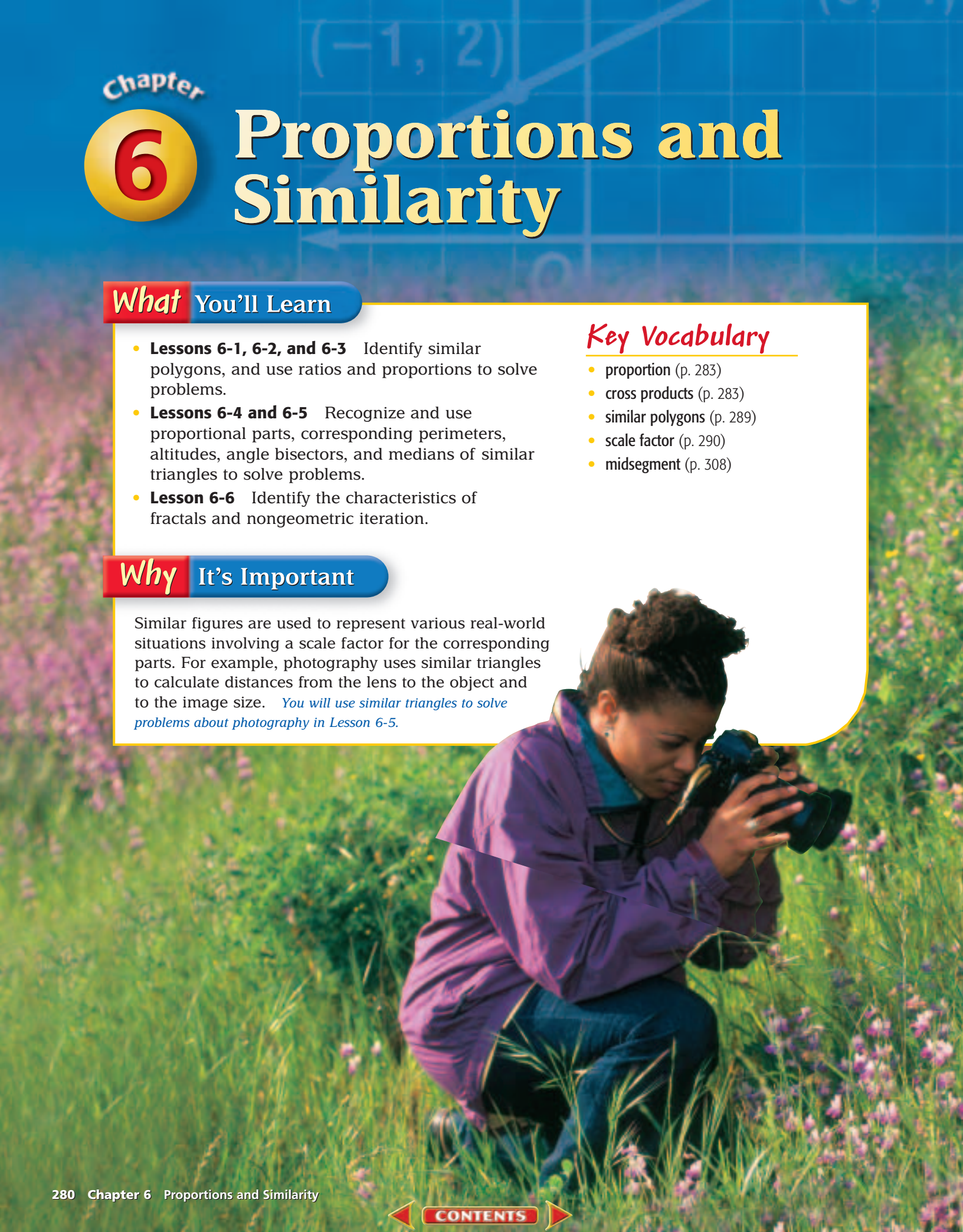
- **Lessons 6-1, 6-2, and 6-3** Identify similar polygons, and use ratios and proportions to solve problems.
- **Lessons 6-4 and 6-5** Recognize and use proportional parts, corresponding perimeters, altitudes, angle bisectors, and medians of similar triangles to solve problems.
- **Lesson 6-6** Identify the characteristics of fractals and nongeometric iteration.

Key Vocabulary

- **proportion** (p. 283)
- **cross products** (p. 283)
- **similar polygons** (p. 289)
- **scale factor** (p. 290)
- **midsegment** (p. 308)

Why It's Important

Similar figures are used to represent various real-world situations involving a scale factor for the corresponding parts. For example, photography uses similar triangles to calculate distances from the lens to the object and to the image size. *You will use similar triangles to solve problems about photography in Lesson 6-5.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lesson 6-1, 6-3, and 6-4 — Solve Rational Equations

Solve each equation. *(For review, see pages 737 and 738.)*

1. $\frac{2}{3}y - 4 = 6$ **2.** $\frac{5}{6} = \frac{x-4}{12}$ **3.** $\frac{4}{3} = \frac{y+2}{y-1}$ **4.** $\frac{2y}{4} = \frac{32}{y}$

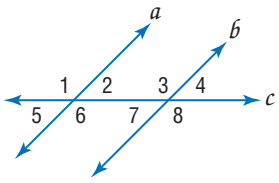
For Lesson 6-2 — Slopes of Lines

Find the slope of the line given the coordinates of two points on the line. *(For review, see Lesson 3-3.)*

5. (3, 5) and (0, −1) **6.** (−6, −3) and (2, −3) **7.** (−3, 4) and (2, −2)

For Lesson 6-5 — Show Lines Parallel

Given the following information, determine whether $a \parallel b$. State the postulate or theorem that justifies your answer. *(For review, see Lesson 3-5.)*



8. $\angle 1 \cong \angle 8$

9. $\angle 3 \cong \angle 6$

10. $\angle 5 \cong \angle 3$

For Lesson 6-6 — Evaluate Expressions

Evaluate each expression for $n = 1, 2, 3$, and 4. *(For review, see page 736.)*

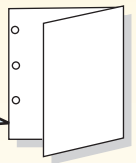
11. 2^n **12.** $n^2 - 2$ **13.** $3^n - 2$

FOLDABLES™ Study Organizer

Proportions and Similarity Make this Foldable to record information about proportions and similarity in this chapter. Begin with one sheet of 11-inch × 17-inch paper.

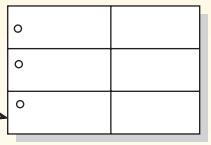
Step 1 Punch and Fold

Fold lengthwise. Leave space to punch holes so it can be placed in your binder.



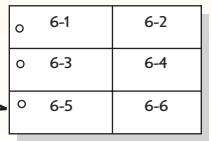
Step 2 Divide

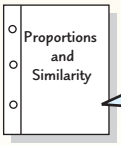
Open the flap and draw lines to divide the inside into six equal parts.



Step 3 Label

Label each part using the lesson numbers.





Put the name of the chapter on the front flap.

Reading and Writing As you read and study the chapter, use the Foldable to write down questions you have about the concepts in each lesson. Leave room to record the answers to your questions.

6-1 Proportions

What You'll Learn

- Write ratios.
- Use properties of proportions.

How do artists use ratios?

Stained-glass artist Louis Comfort Tiffany used geometric shapes in his designs. In a portion of *Clematis Skylight* shown at the right, rectangular shapes are used as the background for the flowers and vines. Tiffany also used ratio and proportion in the design of this piece.

Vocabulary

- ratio
- proportion
- cross products
- extremes
- means

WRITE RATIOS A **ratio** is a comparison of two quantities. The ratio of a to b can be expressed $\frac{a}{b}$, where b is not zero. This ratio can also be written as a:b.

Study Tip

Unit Ratios
Ratios should be written in simplest form. A ratio in which the denominator is 1 is called a *unit ratio*.

Example 1 Write a Ratio

SOCCER **The U.S. Census Bureau surveyed 8218 schools nationally about their girls' soccer programs. They found that 270,273 girls participated in a high school soccer program in the 1999–2000 school year. Find the ratio of girl soccer players per school to the nearest tenth.**

To find this ratio, divide the number of girl athletes by the number of schools.

$$\frac{\text{number of girl soccer players}}{\text{number of schools}} = \frac{270{,}273}{8{,}218} \text{ or about } 32.9$$

32.9 can be written as $\frac{32.9}{1}$. So, the ratio for this survey was 32.9 girl soccer players for each school in the survey.

Extended ratios can be used to compare three or more numbers. The expression a:b:c means that the ratio of the first two numbers is a:b, the ratio of the last two numbers is b:c, and the ratio of the first and last numbers is a:c.

Standardized Test Practice

A B C D

Example 2 Extended Ratios in Triangles

Multiple-Choice Test Item

In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.

(A) 10 in. (B) 60 in. (C) 90 in. (D) 100 in.

Read the Test Item

You are asked to apply the ratio to the three sides of the triangle and the perimeter to find the longest side.

Christie's Images

CONTENTS

The Princeton Review

Test-Taking Tip

Extended ratio problems require a variable to be the common factor among the terms of the ratio. This will enable you to write an equation to solve the problem.

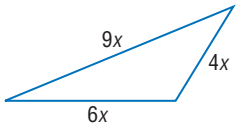
Solve the Test Item

Recall that equivalent fractions can be found by multiplying the numerator and the denominator by the same number. So, $2:3 = \frac{2}{3} \cdot \frac{x}{x}$ or $\frac{2x}{3x}$. Thus, we can rewrite $4:6:9$ as $4x:6x:9x$ and use those measures for the sides of the triangle. Write an equation to represent the perimeter of the triangle as the sum of the measures of its sides.

$4x + 6x + 9x = 190$ Perimeter

$19x = 190$ Combine like terms.

$x = 10$ Divide each side by 19.



Use this value of x to find the measures of the sides of the triangle.

$4x = 4(10)$ or 40 inches

$6x = 6(10)$ or 60 inches

$9x = 9(10)$ or 90 inches

The longest side is 90 inches. The answer is C.

CHECK Add the lengths of the sides to make sure that the perimeter is 190.

$40 + 60 + 90 = 190$ ✓

Study Tip

Reading Mathematics

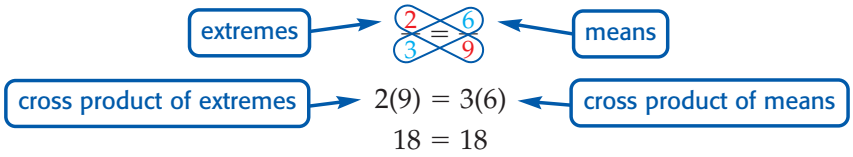
When a proportion is written using colons, it is read using the word *to* for the colon. For example, 2:3 is read *2 to 3*. The means are the inside numbers, and the extremes are the outside numbers.

extremes
2:3 = 6:9
means

USE PROPERTIES OF PROPORTIONS

An equation stating that two ratios are equal is called a **proportion**. Equivalent fractions set equal to each other form a proportion. Since $\frac{2}{3}$ and $\frac{6}{9}$ are equivalent fractions, $\frac{2}{3} = \frac{6}{9}$ is a proportion.

Every proportion has two **cross products**. The cross products in $\frac{2}{3} = \frac{6}{9}$ are 2 times 9 and 3 times 6. The **extremes** of the proportion are 2 and 9. The **means** are 3 and 6.



The product of the means equals the product of the extremes, so the cross products are equal. Consider the general case.

$\frac{a}{b} = \frac{c}{d}$ $b \neq 0, d \neq 0$

$(bd)\frac{a}{b} = (bd)\frac{c}{d}$ Multiply each side by the common denominator, bd.

$da = bc$ Simplify.

$ad = bc$ Commutative Property

Key Concept — Property of Proportions

- **Words** For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
- **Examples** $\frac{4}{5} = \frac{12}{15}$ if and only if $4 \cdot 15 = 5 \cdot 12$.

To *solve a proportion* means to find the value of the variable that makes the proportion true.

Example 3 Solve Proportions by Using Cross Products

Solve each proportion.

a. $\frac{3}{5} = \frac{x}{75}$

$\frac{3}{5} = \frac{x}{75}$		Original proportion
$3(75) = 5x$		Cross products
$225 = 5x$		Multiply.
$45 = x$		Divide each side by 5.

b. $\frac{3x - 5}{4} = \frac{-13}{2}$

$\frac{3x - 5}{4} = \frac{-13}{2}$		Original proportion
$(3x - 5)2 = 4(-13)$		Cross products
$6x - 10 = -52$		Simplify.
$6x = -42$		Add 10 to each side.
$x = -7$		Divide each side by 6.

Proportions can be used to solve problems involving two objects that are said to be *in proportion*. This means that if you write ratios comparing the measures of all parts of one object with the measures of comparable parts of the other object, a true proportion would always exist.

Study Tip

Common Misconception

The proportion shown in Example 4 is not the only correct proportion. There are four equivalent proportions:

$\frac{a}{b} = \frac{c}{d}$, $\frac{a}{d} = \frac{c}{b}$, $\frac{b}{a} = \frac{d}{c}$, and $\frac{b}{c} = \frac{d}{a}$.

All of these have identical cross products.

Example 4 Solve Problems Using Proportions

AVIATION **A twinjet airplane has a length of 78 meters and a wingspan of 90 meters. A toy model is made in proportion to the real airplane. If the wingspan of the toy is 36 centimeters, find the length of the toy.**

Because the toy airplane and the real plane are in proportion, you can write a proportion to show the relationship between their measures. Since both ratios compare meters to centimeters, you need not convert all the lengths to the same unit of measure.

$$\frac{\text{plane's length (m)}}{\text{model's length (cm)}} = \frac{\text{plane's wingspan (m)}}{\text{model's wingspan (cm)}}$$

$\frac{78}{x} = \frac{90}{36}$		Substitution
$(78)(36) = x \cdot 90$		Cross products
$2808 = 90x$		Multiply.
$31.2 = x$		Divide each side by 90.

The length of the model would be 31.2 centimeters.

Check for Understanding

Concept Check

1. **Explain** how you would solve $\frac{28}{48} = \frac{21}{x}$.
2. **OPEN ENDED** Write two possible proportions having the extremes 5 and 8.
3. **FIND THE ERROR** Madeline and Suki are solving $\frac{15}{x} = \frac{3}{4}$.

Madeline

$\frac{15}{x} = \frac{3}{4}$

$45 = 4x$

$11.25 = x$

Suki

$\frac{15}{x} = \frac{3}{4}$

$60 = 3x$

$20 = x$

Who is correct? Explain your reasoning.

Guided Practice

4. **HOCKEY** A hockey player scored 9 goals in 12 games. Find the ratio of goals to games.

5. **SCULPTURE** A replica of *The Thinker* is 10 inches tall. A statue of *The Thinker*, located in front of Grawemeyer Hall on the Belnap Campus of the University of Louisville in Kentucky, is 10 feet tall. What is the ratio of the replica to the statue in Louisville?

Solve each proportion.

6. $\frac{x}{5} = \frac{11}{35}$

7. $\frac{2.3}{4} = \frac{x}{3.7}$

8. $\frac{x-2}{2} = \frac{4}{5}$

9. The ratio of the measures of three sides of a triangle is 9:8:7, and its perimeter is 144 units. Find the measure of each side of the triangle.

10. The ratio of the measures of three angles of a triangle 5:7:8. Find the measure of each angle of the triangle.

Standardized Test Practice

A B C D

11. **GRID IN** The scale on a map indicates that 1.5 centimeters represent 200 miles. If the distance on the map between Norfolk, Virginia, and Atlanta, Georgia, measures 2.4 centimeters, how many miles apart are the cities?

Practice and Apply

Homework Help

For Exercises	See Examples
12–17, 23, 25	1
18–22	2
26, 27	4
28–35	3

Extra Practice
See page 764.

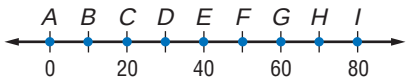
12. **BASEBALL** A designated hitter made 8 hits in 10 games. Find the ratio of hits to games.

13. **SCHOOL** There are 76 boys in a sophomore class of 165 students. Find the ratio of boys to girls.

14. **CURRENCY** In a recent month, 208 South African rands were equivalent to 18 United States dollars. Find the ratio of rands to dollars.

15. **EDUCATION** In the 2000–2001 school year, Arizona State University had 44,125 students and 1747 full-time faculty members. What was the ratio of the students to each teacher rounded to the nearest tenth?

16. Use the number line at the right to determine the ratio of AC to BH.



17. A cable that is 42 feet long is divided into lengths in the ratio of 3:4. What are the two lengths into which the cable is divided?

Find the measures of the angles of each triangle.

18. The ratio of the measures of the three angles is 2:5:3.

19. The ratio of the measures of the three angles is 6:9:10.

Find the measures of the sides of each triangle.

20. The ratio of the measures of three sides of a triangle is 8:7:5. Its perimeter is 240 feet.

21. The ratio of the measures of the sides of a triangle is 3:4:5. Its perimeter is 72 inches.

22. The ratio of the measures of three sides of a triangle are $\frac{1}{2}:\frac{1}{3}:\frac{1}{5}$, and its perimeter is 6.2 centimeters. Find the measure of each side of the triangle.

Courtesy University of Louisville
CONTENTS

LITERATURE **For Exercises 23 and 24, use the following information.**
Throughout Lewis Carroll's book, *Alice's Adventures in Wonderland*, Alice's size changes. Her normal height is about 50 inches tall. She comes across a door, about 15 inches high, that leads to a garden. Alice's height changes to 10 inches so she can visit the garden.

23. Find the ratio of the height of the door to Alice's height in Wonderland.

24. How tall would the door have been in Alice's normal world?

25. **ENTERTAINMENT** Before actual construction of the Great Moments with Mr. Lincoln exhibit, Walt Disney and his design company built models that were in proportion to the displays they planned to build. What is the ratio of the height of the model of Mr. Lincoln compared to his actual height?

More About...

Entertainment

In the model, Lincoln is 8 inches tall. In the theater, Lincoln is 6 feet 4 inches tall (his actual adult height).

Source: Disney

ICE CREAM **For Exercises 26 and 27, use the following information.**
There were approximately 255,082,000 people in the United States in a recent year. According to figures from the United States Census, they consumed about 4,183,344,800 pounds of ice cream that year.

26. If there were 276,000 people in the city of Raleigh, North Carolina, about how much ice cream might they have been expected to consume?

27. Find the approximate consumption of ice cream per person.

Online Research **Data Update** Use the Internet or other resource to find the population of your community. Determine how much ice cream you could expect to be consumed each year in your community. Visit www.geometryonline.com/data_update to learn more.

ALGEBRA **Solve each proportion.**

28. $\frac{3}{8} = \frac{x}{5}$ **29.** $\frac{a}{5.18} = \frac{1}{4}$ **30.** $\frac{3x}{23} = \frac{48}{92}$ **31.** $\frac{13}{49} = \frac{26}{7x}$

32. $\frac{2x - 13}{28} = \frac{-4}{7}$ **33.** $\frac{4x + 3}{12} = \frac{5}{4}$ **34.** $\frac{b + 1}{b - 1} = \frac{5}{6}$ **35.** $\frac{3x - 1}{2} = \frac{-2}{x + 2}$

PHOTOGRAPHY **For Exercises 36 and 37, use the following information.**
José reduced a photograph that is 21.3 centimeters by 27.5 centimeters so that it would fit in a 10-centimeter by 10-centimeter area.

36. Find the maximum dimensions of the reduced photograph.

37. What percent of the original length is the length of the reduced photograph?

38. **CRITICAL THINKING** The ratios of the lengths of the sides of three polygons are given below. Make a conjecture about identifying each type of polygon.

a. 2:2:3 **b.** 3:3:3:3 **c.** 4:5:4:5

39. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do artists use ratios?

Include the following in your answer:

- four rectangles from the photo that appear to be in proportion, and
- an estimate in inches of the ratio of the width of the skylight to the length of the skylight given that the dimensions of the rectangle in the bottom left corner are approximately 3.5 inches by 5.5 inches.

Walt Disney Co.

CONTENTS

Standardized Test Practice

40. SHORT RESPONSE In a golden rectangle, the ratio of the length of the rectangle to its width is approximately 1.618:1. Suppose a golden rectangle has a length of 12 centimeters. What is its width to the nearest tenth?

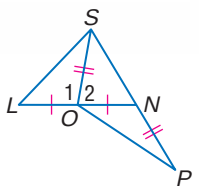
41. ALGEBRA A breakfast cereal contains wheat, rice, and oats in the ratio 3:1:2. If the manufacturer makes a mixture using 120 pounds of oats, how many pounds of wheat will be used?

(A) 60 lb (B) 80 lb (C) 120 lb (D) 180 lb

Maintain Your Skills

Mixed Review

In the figure, $\overline{SO}$ is a median of $\triangle SLN$, $\overline{OS} \cong \overline{NP}$, $m\angle 1 = 3x - 50$, and $m\angle 2 = x + 30$. Determine whether each statement is *always*, *sometimes*, or *never* true. *(Lesson 5-5)*



42. $LS > SN$

43. $SN < OP$

44. $x = 45$

Find the range for the measure of the third side of a triangle given the measures of two sides. *(Lesson 5-4)*

45. 16 and 31 **46.** 26 and 40 **47.** 11 and 23

48. COORDINATE GEOMETRY Given $\triangle STU$ with vertices $S(0, 5)$, $T(0, 0)$, and $U(-2, 0)$ and $\triangle XYZ$ with vertices $X(4, 8)$, $Y(4, 3)$, and $Z(6, 3)$, show that $\triangle STU \cong \triangle XYZ$. *(Lesson 4-4)*

Graph the line that satisfies each condition. *(Lesson 3-3)*

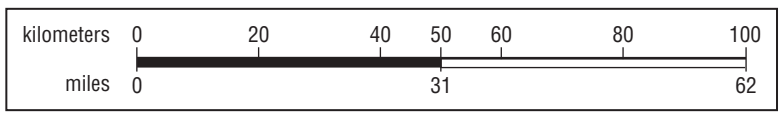
49. $m = \frac{3}{5}$ and contains $P(-3, -4)$

50. contains $A(5, 3)$ and $B(-1, 8)$

51. parallel to $\overleftrightarrow{JK}$ with $J(-1, 5)$ and $K(4, 3)$ and contains $E(2, 2)$

52. contains $S(8, 1)$ and is perpendicular to $\overleftrightarrow{QR}$ with $Q(6, 2)$ and $R(-4, -6)$

53. MAPS On a U.S. map, there is a scale that lists kilometers on the top and miles on the bottom.



Suppose $\overline{AB}$ and $\overline{CD}$ are segments on this map. If $AB = 100$ kilometers and $CD = 62$ miles, is $\overline{AB} \cong \overline{CD}$? Explain. *(Lesson 2-7)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the distance between each pair of points to the nearest tenth.** *(To review the **Distance Formula**, see Lesson 1-3.)*

54. $A(12, 3)$, $B(-8, 3)$ **55.** $C(0, 0)$, $D(5, 12)$

56. $E\left(\frac{4}{5}, -1\right)$, $F\left(2, \frac{-1}{2}\right)$ **57.** $G\left(3, \frac{3}{7}\right)$, $H\left(4, -\frac{2}{7}\right)$

Spreadsheet Investigation

A Follow-Up of Lesson 6-1

Fibonacci Sequence and Ratios

The Fibonacci sequence is a set of numbers that begins with 1 as its first and second terms. Each successive term is the sum of the two numbers before it. This sequence continues on indefinitely.

term	1	2	3	4	5	6	7
Fibonacci number	1	1	2	3	5	8	13
			↑	↑	↑	↑	↑
			1 + 1	1 + 2	2 + 3	3 + 5	5 + 8

Example

Use a spreadsheet to create twenty terms of the Fibonacci sequence. Then compare each term with its preceding term.

Step 1 Enter the column headings in rows 1 and 2.

Step 2 Enter 1 into cell A3. Then insert the formula =A3 + 1 in cell A4. Copy this formula down the column. This will automatically calculate the number of the term.

Step 3 In column B, we will record the Fibonacci numbers. Enter 1 in cells B3 and B4 since you do not have two previous terms to add. Then insert the formula =B3 + B4 in cell B5. Copy this formula down the column.

Step 4 In column C, we will find the ratio of each term to its preceding term. Enter 1 in cell C3 since there is no preceding term. Then enter =B4/B3 in cell C4. Copy this formula down the column.

Fibonacci Table

	A	B	C
1	term	Fibonacci number	ratio
2	n	F(n)	F(n+1)/F(n)
3	1	1	1
4	2	1	1
5	3	2	2
6	4	3	1.5
7	5	5	1.666666667
8	6	8	1.6
9	7	13	1.625

Sheet1 Sheet2

Exercises

1. What happens to the Fibonacci number as the number of the term increases?
2. What pattern of odd and even numbers do you notice in the Fibonacci sequence?
3. As the number of terms gets greater, what pattern do you notice in the ratio column?
4. Extend the spreadsheet to calculate fifty terms of the Fibonacci sequence. Describe any differences in the patterns you described in Exercises 1–3.

The rectangle that most humans perceive to be pleasing to the eye has a width to length ratio of about 1:1.618. This is called the *golden ratio*, and the rectangle is called the *golden rectangle*. This type of rectangle is visible in nature and architecture. The Fibonacci sequence occurs in nature in patterns that are also pleasing to the human eye, such as in sunflowers, pineapples, and tree branch structure.

5. **MAKE A CONJECTURE** How might the Fibonacci sequence relate to the golden ratio?

6-2 Similar Polygons

What You'll Learn

- Identify similar figures.
- Solve problems involving scale factors.

Vocabulary

- similar polygons
- scale factor

How do artists use geometric patterns?

M.C. Escher (1898–1972) was a Dutch graphic artist known for drawing impossible structures, spatial illusions, and repeating interlocking geometric patterns. The image at the right is a print of Escher's *Circle Limit IV*, which is actually a woodcutting. It includes winged images that have the same shape, but are different in size. Also note that there are not only similar dark images but also similar light images.

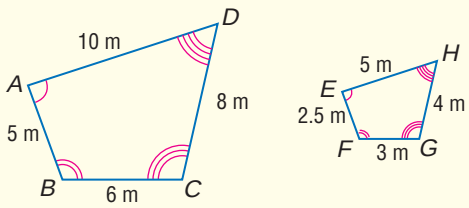
Circle Limit IV, M.C. Escher (1960)

©2002 Cordon Art B.V., Baarn, Holland. All rights reserved.

IDENTIFY SIMILAR FIGURES When polygons have the same shape but may be different in size, they are called **similar polygons**.

Key Concept — Similar Polygons

- **Words** Two polygons are similar if and only if their corresponding angles are congruent and the measures of their corresponding sides are proportional.
- **Symbol** $\sim$ is read *is similar to*
- **Example**



Study Tip

Similarity and Congruence
If two polygons are congruent, they are also similar. All of the corresponding angles are congruent, and the lengths of the corresponding sides have a ratio of 1:1.

The order of the vertices in a similarity statement is important. It identifies the corresponding angles and the corresponding sides.

similarity statement	congruent angles	corresponding sides
$ABCD \sim EFGH$	$\angle A \cong \angle E$ $\angle B \cong \angle F$ $\angle C \cong \angle G$ $\angle D \cong \angle H$	$\frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{DA}{HE}$

Like congruent polygons, similar polygons may be repositioned so that corresponding parts are easy to identify.

Art Resource, NY

CONTENTS

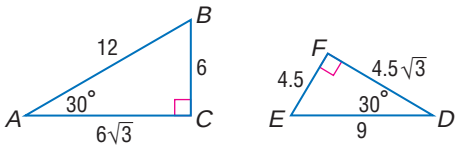
Example 1 Similar Polygons

Study Tip

Identifying Corresponding Parts
Using different colors to circle letters of congruent angles may help you identify corresponding parts.

Determine whether each pair of figures is similar. Justify your answer.

a.



All right angles are congruent, so $\angle C \cong \angle F$.
Since $m\angle A = m\angle D$, $\angle A \cong \angle D$.
By the Third Angle Theorem, $\angle B \cong \angle E$.
Thus, all corresponding angles are congruent.

Now determine whether corresponding sides are proportional.

Sides opposite 90° angle: $\frac{AB}{DE} = \frac{12}{9}$ or $1.\overline{3}$

Sides opposite 30° angle: $\frac{BC}{EF} = \frac{6}{4.5}$ or $1.\overline{3}$

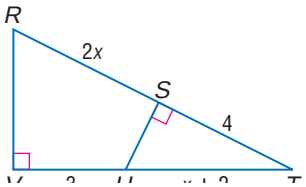
Sides opposite 60° angle: $\frac{AC}{DF} = \frac{6\sqrt{3}}{4.5\sqrt{3}}$ or $1.\overline{3}$

The ratios of the measures of the corresponding sides are equal, and the corresponding angles are congruent, so $\triangle ABC \sim \triangle DEF$.

Study Tip

Common Misconception
When two figures have vertices that are in alphabetical order, this does not mean that the corresponding vertices in the similarity statement will follow alphabetical order.

b.



Both rectangles have all right angles and right angles are congruent.
$\frac{AB}{EF} = \frac{7}{6}$ and $\frac{BC}{FG} = \frac{6}{5}$, but $\frac{AB}{EF} \neq \frac{BC}{FG}$ because $\frac{7}{6} \neq \frac{6}{5}$. The rectangles are not similar.

SCALE FACTORS When you compare the lengths of corresponding sides of similar figures, you usually get a numerical ratio. This ratio is called the **scale factor** for the two figures. Scale factors are often given for models of real-life objects.

Example 2 Scale Factor

MOVIES **Some special effects in movies are created using miniature models. In a recent movie, a model sports-utility vehicle (SUV) 22 inches long was created to look like a real $14\frac{2}{3}$-foot SUV. What is the scale factor of the model compared to the real SUV?**

Before finding the scale factor you must make sure that both measurements use the same unit of measure.

$$14\tfrac{2}{3}(12) = 176 \text{ inches}$$

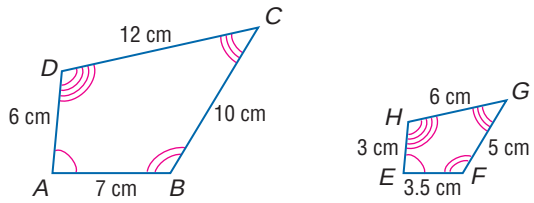
$$\frac{\text{length of model}}{\text{length of real SUV}} = \frac{22 \text{ inches}}{176 \text{ inches}}$$

$$= \frac{1}{8}$$

The ratio comparing the two lengths is $\frac{1}{8}$ or 1:8. The scale factor is $\frac{1}{8}$, which means that the model is $\frac{1}{8}$ the length of the real SUV.

CONTENTS

When finding the scale factor for two similar polygons, the scale factor will depend on the order of comparison.



- The scale factor of quadrilateral $ABCD$ to quadrilateral $EFGH$ is 2.
- The scale factor of quadrilateral $EFGH$ to quadrilateral $ABCD$ is $\frac{1}{2}$.

Example 3 Proportional Parts and Scale Factor

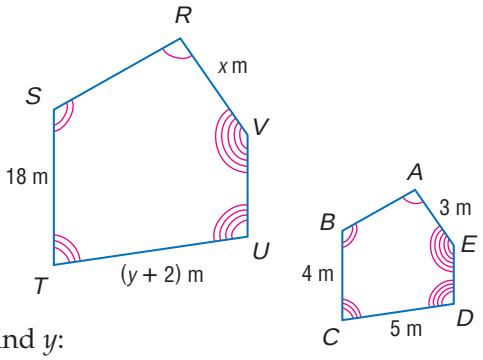
The two polygons are similar.

a. Write a similarity statement. Then find x, y, and UT.

Use the congruent angles to write the corresponding vertices in order.

polygon $RSTUV \sim$ polygon $ABCDE$

Now write proportions to find x and y.



To find x:

$\frac{ST}{BC} = \frac{VR}{EA}$	Similarity proportion
$\frac{18}{4} = \frac{x}{3}$	$ST = 18$, $BC = 4$, $VR = x$, $EA = 3$
$18(3) = 4(x)$	Cross products
$54 = 4x$	Multiply.
$13.5 = x$	Divide each side by 4.

To find y:

$\frac{ST}{BC} = \frac{UT}{DC}$	Similarity proportion
$\frac{18}{4} = \frac{y+2}{5}$	$ST = 18$, $BC = 4$, $UT = y + 2$, $EA = 3$
$18(5) = 4(y + 2)$	Cross products
$90 = 4y + 8$	Multiply.
$82 = 4y$	Subtract 8 from each side.
$20.5 = y$	Divide each side by 4.

$UT = y + 2$, so $UT = 20.5 + 2$ or 22.5.

b. Find the scale factor of polygon *RSTUV* to polygon *ABCDE*.

The scale factor is the ratio of the lengths of any two corresponding sides.

$\frac{ST}{BC} = \frac{18}{4}$ or $\frac{9}{2}$

Study Tip

Checking Solutions

To verify the scale factor, find the ratio of two other corresponding sides.

You can use scale factors to produce similar figures.

Example 4 Enlargement of a Figure

Triangle ABC is similar to $\triangle XYZ$ with a scale factor of $\frac{2}{3}$. If the lengths of the sides of $\triangle ABC$ are 6, 8, and 10 inches, what are the lengths of the sides of $\triangle XYZ$?

Write proportions for finding side measures.

$\triangle ABC \rightarrow \frac{6}{x} = \frac{2}{3}$
$\triangle XYZ \rightarrow$

$18 = 2x$

$9 = x$

$\triangle ABC \rightarrow \frac{8}{y} = \frac{2}{3}$
$\triangle XYZ \rightarrow$

$24 = 2y$

$12 = y$

$\triangle ABC \rightarrow \frac{10}{z} = \frac{2}{3}$
$\triangle XYZ \rightarrow$

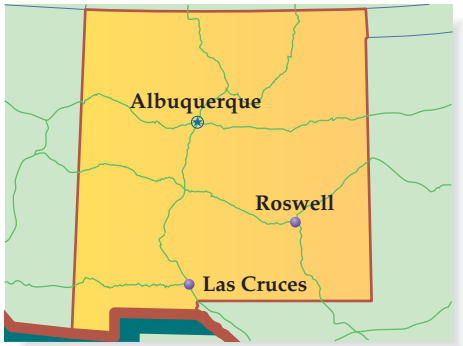
$30 = 2z$

$15 = z$

The lengths of the sides of $\triangle XYZ$ are 9, 12, and 15 inches.

Example 5 Scale Factors on Maps

MAPS **The scale on the map of New Mexico is 2 centimeters = 160 miles. The distance on the map across New Mexico from east to west through Albuquerque is 4.1 centimeters. How long would it take to drive across New Mexico if you drove at an average of 60 miles per hour?**



Explore Every 2 centimeters represents 160 miles. The distance across the map is 4.1 centimeters.

Plan Create a proportion relating the measurements to the scale to find the distance in miles. Then use the formula $d = rt$ to find the time.

Solve

$$\begin{array}{rll} \text{centimeters} \rightarrow \\ \text{miles} \rightarrow \end{array} \quad \frac{2}{160} = \frac{4.1}{x} \quad \begin{array}{l} \leftarrow \text{centimeters} \\ \leftarrow \text{miles} \end{array}$$

$$2x = 656 \quad \text{Cross products}$$

$$x = 328 \quad \text{Divide each side by 2.}$$

The distance across New Mexico is approximately 328 miles.

$$d = rt$$

$$328 = 60t \quad d = 328 \text{ and } r = 60$$

$$\frac{328}{60} = t \quad \text{Divide each side by 60.}$$

$$5\frac{7}{15} = t \quad \text{Simplify.}$$

It would take $5\frac{7}{15}$ hours or about 5 hours and 28 minutes to drive across New Mexico at an average of 60 miles per hour.

Examine To determine whether the answer is reasonable, reexamine the scale. If 2 centimeters = 160 miles, then 4 centimeters = 320 miles. The map is about 4 centimeters wide, so the distance across New Mexico is about 320 miles. The answer is about 5.5 hours and at 60 miles per hour, the trip would be 330 miles. The two distances are close estimates, so the answer is reasonable.

Study Tip

Units of Time

Remember that there are 60 minutes in an hour. When rewriting $\frac{328}{60}$ as a mixed number, you could also write $5\frac{28}{60}$, which means 5 hours 28 minutes.

Check for Understanding

Concept Check

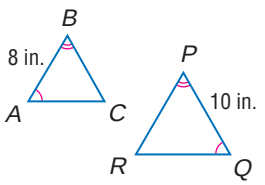
1. **FIND THE ERROR** Roberto and Garrett have calculated their scale factor for two similar triangles.

Roberto

$$\frac{AB}{PQ} = \frac{8}{10} = \frac{4}{5}$$

Garrett

$$\frac{PQ}{AB} = \frac{10}{8} = \frac{5}{4}$$



Who is correct? Explain your reasoning.

CONTENTS

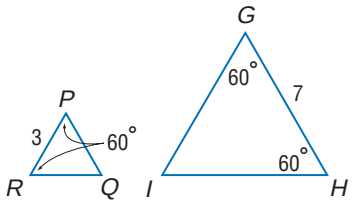
2. **Find a counterexample** for the statement *All rectangles are similar.*

3. **OPEN ENDED** Explain whether two polygons that are congruent are also similar. Then explain whether two polygons that are similar are also congruent.

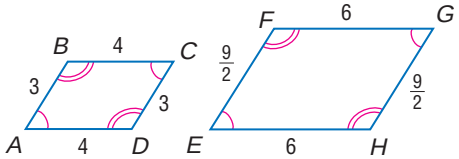
Guided Practice

Determine whether each pair of figures is similar. Justify your answer.

4.

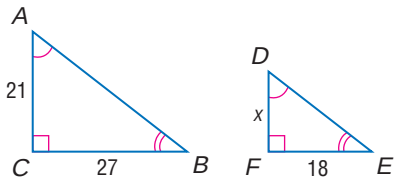


5.

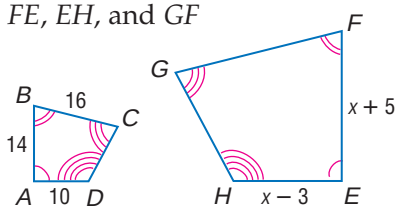


Each pair of polygons is similar. Write a similarity statement, and find x, the measure(s) of the indicated side(s), and the scale factor.

6. $\overline{DF}$



7. $\overline{FE}$, $\overline{EH}$, and $\overline{GF}$



8. A rectangle with length 60 centimeters and height 40 centimeters is reduced so that the new rectangle is similar to the original and the scale factor is $\frac{1}{4}$. Find the length and height of the new rectangle.

9. A triangle has side lengths of 3 meters, 5 meters, and 4 meters. The triangle is enlarged so that the larger triangle is similar to the original and the scale factor is 5. Find the perimeter of the larger triangle.

Application

10. **MAPS** Refer to Example 5 on page 292. Draw the state of New Mexico using a scale of 2 centimeters = 100 miles. Is your drawing similar to the one in Example 4? Explain how you know.

Practice and Apply

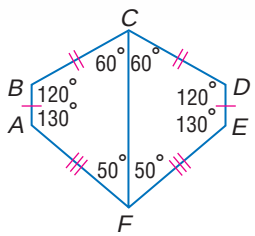
Homework Help

For Exercises	See Examples
11–14	1
15–20, 34–39	2, 3
21–23	4
24–26	5

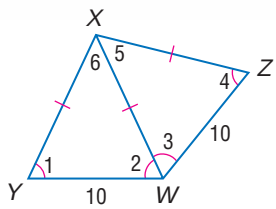
Extra Practice
See page 765.

Determine whether each pair of figures is similar. Justify your answer.

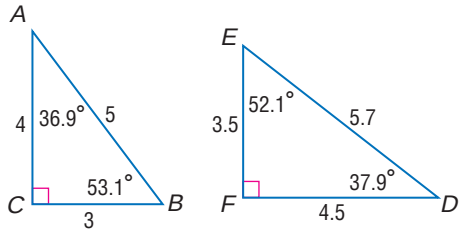
11.



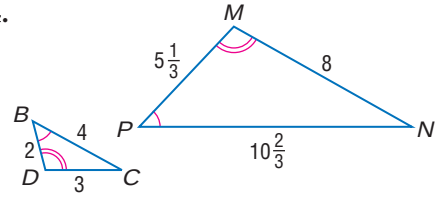
12.



13.



14.



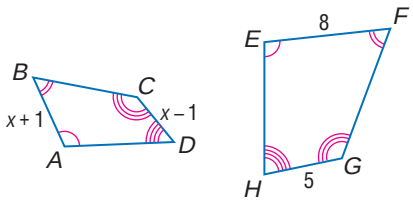
CONTENTS

15. **ARCHITECTURE** The replica of the Eiffel Tower at an amusement park is $350\frac{2}{3}$ feet tall. The actual Eiffel Tower is 1052 feet tall. What is the scale factor comparing the amusement park tower to the actual tower?

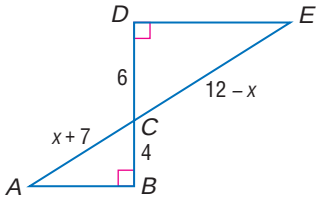
16. **PHOTOCOPYING** Mr. Richardson walked to a copier in his office, made a copy of his proposal, and sent the original to one of his customers. When Mr. Richardson looked at his copy before filing it, he saw that the copy had been made at an 80% reduction. He needs his filing copy to be the same size as the original. What enlargement scale factor must he use on the first copy to make a second copy the same size as the original?

Each pair of polygons is similar. Write a similarity statement, and find x, the measures of the indicated sides, and the scale factor.

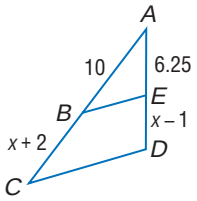
17. $\overline{AB}$ and $\overline{CD}$



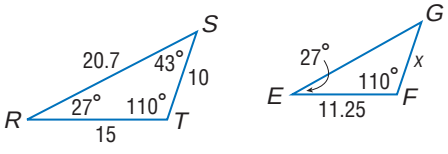
18. $\overline{AC}$ and $\overline{CE}$



19. $\overline{BC}$ and $\overline{ED}$



20. $\overline{GF}$ and $\overline{EG}$



PHOTOGRAPHY **For Exercises 21–23, use the following information.**

A picture is enlarged by a scale factor of $\frac{5}{4}$ and then enlarged again by the same factor.

21. If the original picture was 2.5 inches by 4 inches, what were its dimensions after both enlargements?
22. Write an equation describing the enlargement process.
23. By what scale factor was the original picture enlarged?

SPORTS **Make a scale drawing of each playing field using the given scale.**

24. Use the information about the soccer field in Crew Stadium. Use the scale 1 millimeter = 1 meter.
25. A basketball court is 84 feet by 50 feet. Use the scale $\frac{1}{4}$ inch = 4 feet.
26. A tennis court is 36 feet by 78 feet. Use the scale $\frac{1}{8}$ inch = 1 foot.

Determine whether each statement is *always*, *sometimes*, or *never* true.

27. Two congruent triangles are similar.
28. Two squares are similar.
29. A triangle is similar to a quadrilateral.
30. Two isosceles triangles are similar.
31. Two rectangles are similar.
32. Two obtuse triangles are similar.
33. Two equilateral triangles are similar.

More About . . .

Sports

Crew Stadium in Columbus, Ohio, was specifically built for Major League Soccer. The dimensions of the field are about 69 meters by 105 meters.

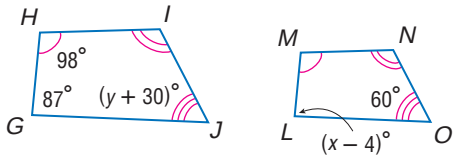
Source: www.MLSnet.com

Joe Giblin/Columbus Crew/MLS

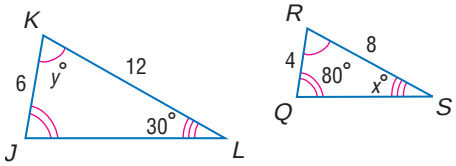
CONTENTS

Each pair of polygons is similar. Find x and y. Round to the nearest hundredth if necessary.

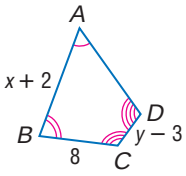
34.

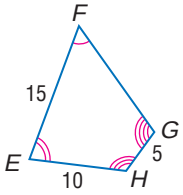


35.

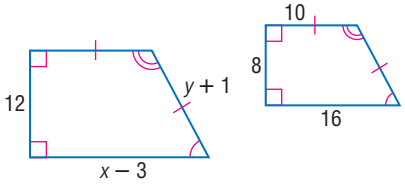


36.

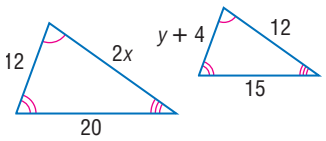




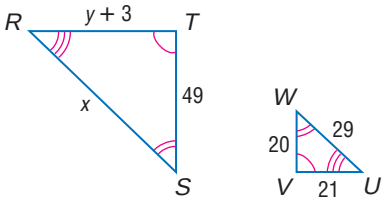
37.



38.



39.



For Exercises 40–47, use the following information to find each measure.
Polygon $ABCD \sim$ polygon $AEFG$, $m\angle AGF = 108$, $GF = 14$, $AD = 12$, $DG = 4.5$, $EF = 8$, and $AB = 26$.

40. scale factor of trapezoid $ABCD$ to trapezoid $AEFG$

41. AG

42. DC

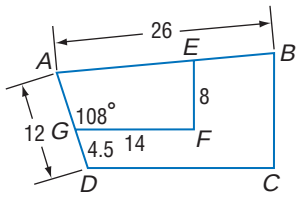
43. $m\angle ADC$

44. BC

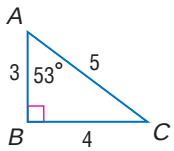
45. perimeter of trapezoid $ABCD$

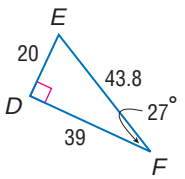
46. perimeter of trapezoid $AEFG$

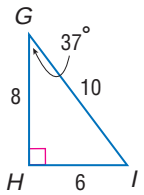
47. ratio of the perimeter of polygon $ABCD$ to the perimeter of polygon $AEFG$

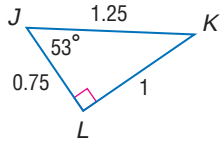


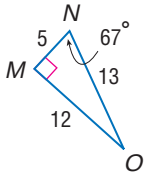
48. Determine which of the following right triangles are similar. Justify your answer.

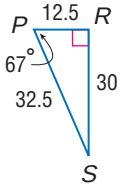










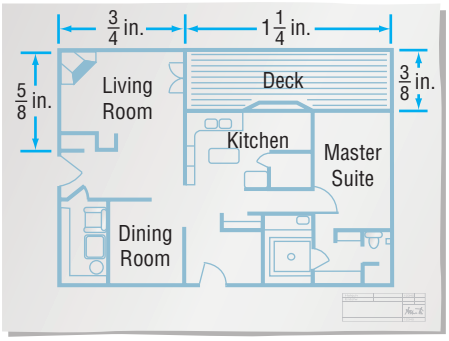


COORDINATE GEOMETRY **Graph the given points. Draw polygon $ABCD$ and $\overline{MN}$. Find the coordinates for vertices L and P such that $ABCD \sim NLPM$.**

49. $A(2, 0)$, $B(4, 4)$, $C(0, 4)$, $D(-2, 0)$; $M(4, 0)$, $N(12, 0)$

50. $A(-7, 1)$, $B(2, 5)$, $C(7, 0)$, $D(-2, -4)$; $M(-3, 1)$, $N\left(-\frac{11}{2}, \frac{7}{2}\right)$

CONSTRUCTION **For Exercises 51 and 52, use the following information.**
A floor plan is given for the first floor of a new house. One inch represents 24 feet. Use the information in the plan to find the dimensions.



51. living room

52. deck

CRITICAL THINKING **For Exercises 53–55, use the following information.**
The area A of a rectangle is the product of its length ℓ and width w. Rectangle $ABCD$ is similar to rectangle $WXYZ$ with sides in a ratio of 4:1.

53. What is the ratio of the areas of the two rectangles?

54. Suppose the dimension of each rectangle is tripled. What is the new ratio of the sides of the rectangles?

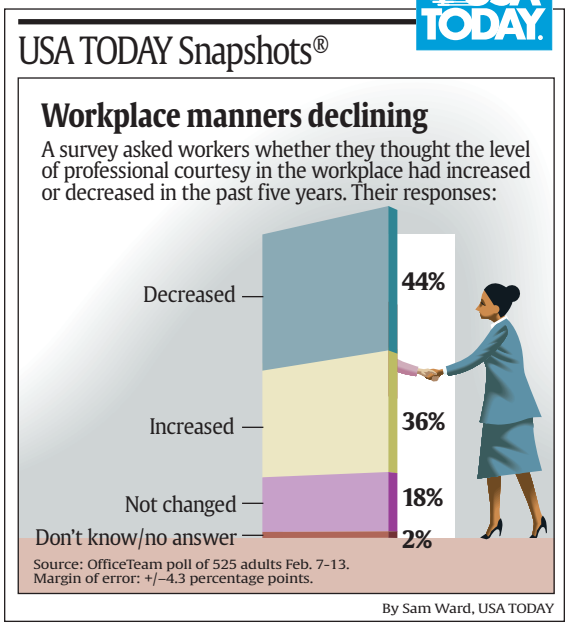
55. What is the ratio of the areas of these larger rectangles?

STATISTICS **For Exercises 56–58, refer to the graphic, which uses rectangles to represent percents.**

56. Are the rectangles representing 36% and 18% similar? Explain.

57. What is the ratio of the areas of the rectangles representing 36% and 18% if area = length × width? Compare the ratio of the areas to the ratio of the percents.

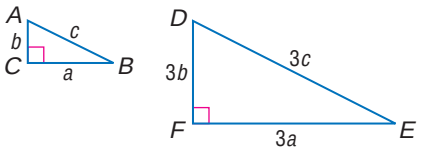
58. Use the graph to make a conjecture about the overall changes in the level of professional courtesy in the workplace in the past five years.



CRITICAL THINKING **For Exercises 59 and 60, $\triangle ABC \sim \triangle DEF$.**

59. Show that the perimeters of $\triangle ABC$ and $\triangle DEF$ have the same ratio as their corresponding sides.

60. If 6 units are added to the lengths of each side, are the new triangles similar?



61. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do artists use geometric patterns?

Include the following in your answer:

- why Escher called the picture *Circle Limit Four*, and
- how one of the light objects and one of the dark objects compare in size.

CONTENTS

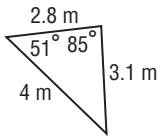
Standardized Test Practice

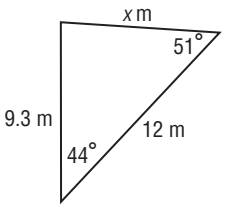
62. In a history class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

(A) 2 (B) 8 (C) 12 (D) 15

63. **ALGEBRA** Find x.

(A) 4.2 (B) 4.65
(C) 5.6 (D) 8.4





Extending the Lesson

Scale factors can be used to produce similar figures. The resulting figure is an enlargement or reduction of the original figure depending on the scale factor.

Triangle ABC has vertices $A(0, 0)$, $B(8, 0)$, and $C(2, 7)$. Suppose the coordinates of each vertex are multiplied by 2 to create the similar triangle $A'B'C'$.

64. Find the coordinates of the vertices of $\triangle A'B'C'$.

65. Graph $\triangle ABC$ and $\triangle A'B'C'$.

66. Use the Distance Formula to find the measures of the sides of each triangle.

67. Find the ratios of the sides that appear to correspond.

68. How could you use slope to determine if angles are congruent?

69. Is $\triangle ABC \sim \triangle A'B'C'$? Explain your reasoning.

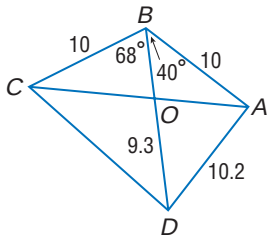
Maintain Your Skills

Mixed Review

Solve each proportion. *(Lesson 6-1)*

70. $\frac{b}{7.8} = \frac{2}{3}$ **71.** $\frac{c-2}{c+3} = \frac{5}{4}$ **72.** $\frac{2}{4y+5} = \frac{-4}{y}$

Use the figure to write an inequality relating each pair of angle or segment measures. *(Lesson 5-5)*



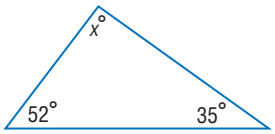
73. OC, AO

74. $m\angle AOD$, $m\angle AOB$

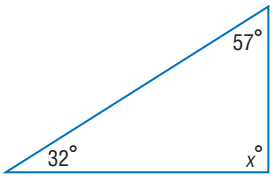
75. $m\angle ABD$, $m\angle ADB$

Find x. *(Lesson 4-2)*

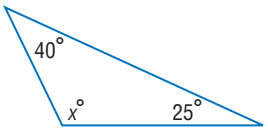
76.



77.



78.



79. Suppose two parallel lines are cut by a transversal and $\angle 1$ and $\angle 2$ are alternate interior angles. Find $m\angle 1$ and $m\angle 2$ if $m\angle 1 = 10x - 9$ and $m\angle 2 = 9x + 3$. *(Lesson 3-2)*

Getting Ready for the Next Lesson

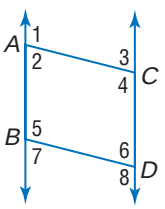
PREREQUISITE SKILL **In the figure, $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \parallel \overline{BD}$, and $m\angle 4 = 118$. Find the measure of each angle.** *(To review **angles and parallel lines**, see Lesson 3-2.)*

80. $\angle 1$ **81.** $\angle 2$

82. $\angle 3$ **83.** $\angle 5$

84. $\angle ABD$ **85.** $\angle 6$

86. $\angle 7$ **87.** $\angle 8$



6-3 Similar Triangles

What You'll Learn

- Identify similar triangles.
- Use similar triangles to solve problems.

How do engineers use geometry?

- The Eiffel Tower was built in Paris for the 1889 world exhibition by Gustave Eiffel. Eiffel (1832–1923) was a French engineer who specialized in revolutionary steel constructions. He used thousands of triangles, some the same shape but different in size, to build the Eiffel Tower because triangular shapes result in rigid construction.

More About . . .

Eiffel Tower

The Eiffel Tower weighs 7000 tons, but the pressure per square inch it applies on the ground is only equivalent to that of a chair with a person seated in it.

Source: www.eiffel-tower.com

IDENTIFY SIMILAR TRIANGLES

In Chapter 4, you learned several tests to determine whether two triangles are congruent. There are also tests to determine whether two triangles are similar.

Geometry Activity

Similar Triangles

Collect Data

- Draw $\triangle DEF$ with $m\angle D = 35$, $m\angle F = 80$, and $DF = 4$ centimeters.
- Draw $\triangle RST$ with $m\angle T = 35$, $m\angle S = 80$, and $ST = 7$ centimeters.
- Measure $\overline{EF}$, $\overline{ED}$, $\overline{RS}$, and $\overline{RT}$.
- Calculate the ratios $\frac{FD}{ST}$, $\frac{EF}{RS}$, and $\frac{ED}{RT}$.

Analyze the Data

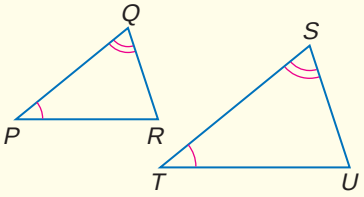
1. What can you conclude about all of the ratios?
2. Repeat the activity with two more triangles with the same angle measures, but different side measures. Then repeat the activity with a third pair of triangles. Are all of the triangles similar? Explain.
3. What are the minimum requirements for two triangles to be similar?

The previous activity leads to the following postulate.

Postulate 6.1

Angle-Angle (AA) Similarity If the two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Example: $\angle P \cong \angle T$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



You can use the AA Similarity Postulate to prove two theorems that also verify triangle similarity.

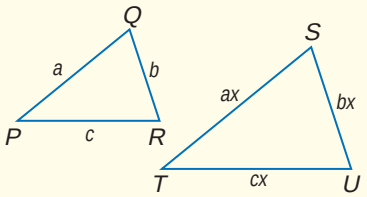
Jeremy Walker/Getty Images

CONTENTS

Theorems

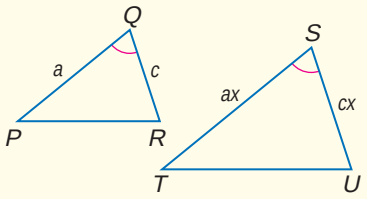
6.1 Side-Side-Side (SSS) Similarity If the measures of the corresponding sides of two triangles are proportional, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU} = \frac{RP}{UT}$, so $\triangle PQR \sim \triangle TSU$.



6.2 Side-Angle-Side (SAS) Similarity If the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar.

Example: $\frac{PQ}{ST} = \frac{QR}{SU}$ and $\angle Q \cong \angle S$, so $\triangle PQR \sim \triangle TSU$.



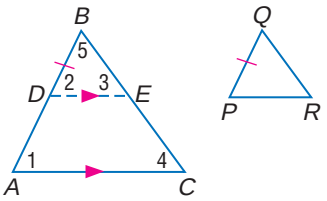
You will prove Theorem 6.2 in Exercise 34.

Proof Theorem 6.1

Given: $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$

Prove: $\triangle BAC \sim \triangle QPR$

Locate D on $\overline{AB}$ so that $\overline{DB} \cong \overline{PQ}$ and draw $\overline{DE}$ so that $\overline{DE} \parallel \overline{AC}$.



Paragraph Proof:

Since $\overline{DB} \cong \overline{PQ}$, $DB = PQ$. $\frac{PQ}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$ becomes $\frac{DB}{AB} = \frac{QR}{BC} = \frac{RP}{CA}$. Since $\overline{DE} \parallel \overline{AC}$, $\angle 2 \cong \angle 1$ and $\angle 3 \cong \angle 4$. By AA Similarity, $\triangle BDE \sim \triangle BAC$.

By the definition of similar polygons, $\frac{DB}{AB} = \frac{BE}{BC} = \frac{ED}{CA}$. By substitution, $\frac{QR}{BC} = \frac{BE}{BC}$ and $\frac{RP}{CA} = \frac{ED}{CA}$. This means that $QR = BE$ and $RP = ED$ or $\overline{QR} \cong \overline{BE}$ and $\overline{RP} \cong \overline{ED}$. With these congruences and $\overline{DB} \cong \overline{PQ}$, $\triangle BDE \cong \triangle QPR$ by SSS. By CPCTC, $\angle B \cong \angle Q$ and $\angle 2 \cong \angle P$. But $\angle 2 \cong \angle A$, so $\angle A \cong \angle P$. By AA Similarity, $\triangle BAC \sim \triangle QPR$.

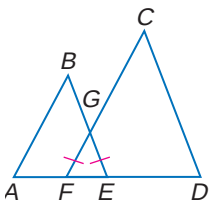
Study Tip

Overlapping Triangles

When two triangles overlap, draw them separately so the corresponding parts are in the same position on the paper. Then write the corresponding angles and sides.

Example 1 Determine Whether Triangles Are Similar

In the figure, $\overline{FG} \cong \overline{EG}$, $BE = 15$, $CF = 20$, $AE = 9$, and $DF = 12$. Determine which triangles in the figure are similar.



Triangle FGE is an isosceles triangle. So, $\angle GFE \cong \angle GEF$. If the measures of the corresponding sides that include the angles are proportional, then the triangles are similar.

$\frac{AE}{DF} = \frac{9}{12}$ or $\frac{3}{4}$ and $\frac{BE}{CF} = \frac{15}{20}$ or $\frac{3}{4}$

By substitution, $\frac{AE}{DF} = \frac{BE}{CF}$. So, by SAS Similarity, $\triangle ABE \sim \triangle DCF$.

Like the congruence of triangles, similarity of triangles is reflexive, symmetric, and transitive.

Theorem 6.3

Similarity of triangles is reflexive, symmetric, and transitive.

Examples:

Reflexive: $\triangle ABC \sim \triangle ABC$

Symmetric: If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$.

Transitive: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHI$, then $\triangle ABC \sim \triangle GHI$.

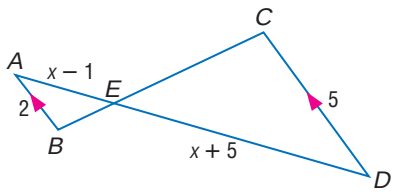
You will prove Theorem 6.3 in Exercise 38.

USE SIMILAR TRIANGLES

Similar triangles can be used to solve problems.

Example 2 Parts of Similar Triangles

ALGEBRA **Find *AE* and *DE*.**



Since $\overline{AB} \parallel \overline{CD}$, $\angle BAE \cong \angle CDE$ and $\angle ABE \cong \angle DCE$ because they are the alternate interior angles. By AA Similarity, $\triangle ABE \sim \triangle DCE$. Using the definition of similar polygons, $\frac{AB}{DC} = \frac{AE}{DE}$.

$$\frac{AB}{DC} = \frac{AE}{DE}$$

$$\frac{2}{5} = \frac{x-1}{x+5} \quad \text{Substitution}$$

$2(x + 5) = 5(x - 1)$ Cross products

$2x + 10 = 5x - 5$ Distributive Property

$-3x = -15$ Subtract $5x$ and 10 from each side.

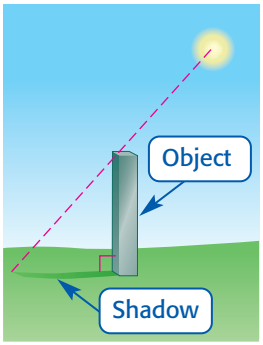
$x = 5$ Divide each side by -3.

Now find AE and ED. $AE = x - 1 = 5 - 1$ or 4 $\quad ED = x + 5 = 5 + 5$ or 10

Study Tip

Shadow Problems

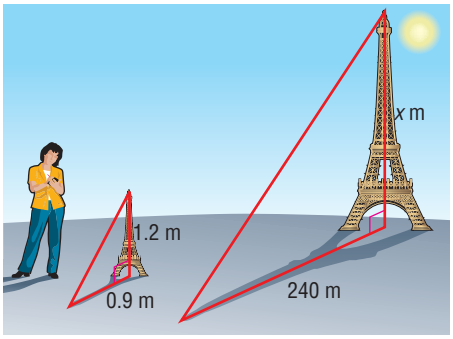
In shadow problems, we assume that a right triangle is formed by the sun's ray from the top of the object to the end of the shadow.



Similar triangles can be used to find measurements indirectly.

Example 3 Find a Measurement

INDIRECT MEASUREMENT **Nina was curious about the height of the Eiffel Tower. She used a 1.2 meter model of the tower and measured its shadow at 2 P.M. The length of the shadow was 0.9 meter. Then she measured the Eiffel Tower's shadow, and it was 240 meters. What is the height of the Eiffel Tower?**



Assuming that the sun's rays form similar triangles, the following proportion can be written.

$$\frac{\text{height of the Eiffel Tower (m)}}{\text{height of the model tower (m)}} = \frac{\text{Eiffel Tower shadow length (m)}}{\text{model shadow length (m)}}$$

CONTENTS

Now substitute the known values and let x be the height of the Eiffel Tower.

$\frac{x}{1.2} = \frac{240}{0.9}$ Substitution

$x \cdot 0.9 = 1.2(240)$ Cross products

$0.9x = 288$ Simplify.

$x = 320$ Divide each side by 0.9.

The Eiffel Tower is 320 meters tall.

Check for Understanding

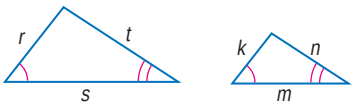
Concept Check

1. **Compare and contrast** the tests to prove triangles similar with the tests to prove triangles congruent.

2. **OPEN ENDED** Is it possible that $\triangle ABC$ is not similar to $\triangle RST$ and that $\triangle RST$ is not similar to $\triangle EFG$, but that $\triangle ABC$ is similar to $\triangle EFG$? Explain.

3. **FIND THE ERROR** Alicia and Jason were writing proportions for the similar triangles shown at the right.

Alicia	Jason
$\frac{r}{k} = \frac{s}{m}$	$\frac{r}{k} = \frac{m}{s}$
$rm = ks$	$rs = km$

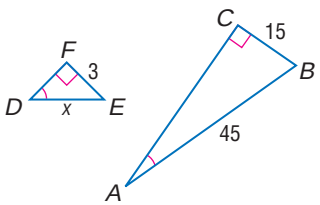


Who is correct? Explain your reasoning.

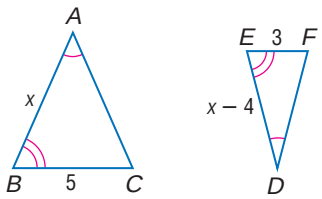
Guided Practice

ALGEBRA **Identify the similar triangles. Find x and the measures of the indicated sides.**

4. DE

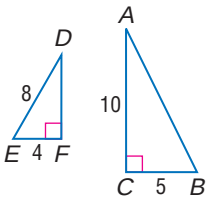


5. AB and DE

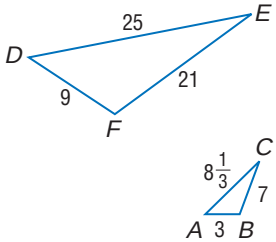


Determine whether each pair of triangles is similar. Justify your answer.

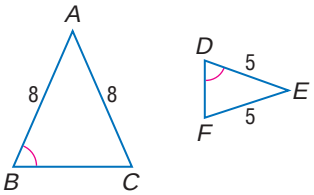
6.



7.



8.



Application

9. **INDIRECT MEASUREMENT** A cell phone tower in a field casts a shadow of 100 feet. At the same time, a 4 foot 6 inch post near the tower casts a shadow of 3 feet 4 inches. Find the height of the tower in feet and inches. (*Hint*: Make a drawing.)

Practice and Apply

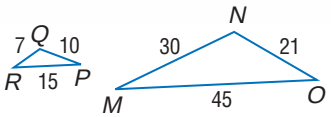
Homework Help

For Exercises	See Examples
10–17, 26–27	1
18–21, 28–31	2
38–41	3

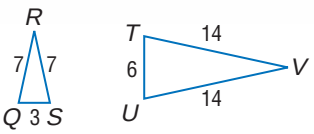
Extra Practice
See page 765.

Determine whether each pair of triangles is similar. Justify your answer.

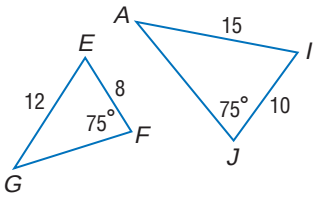
10.



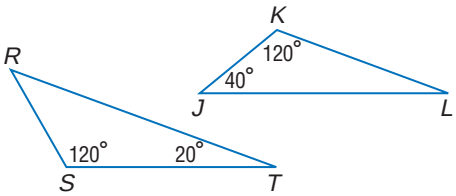
11.



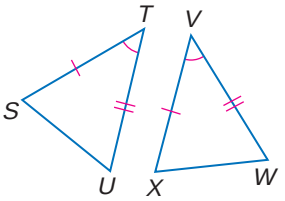
12.



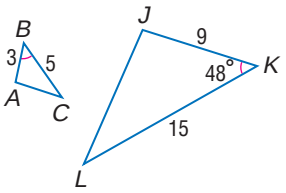
13.



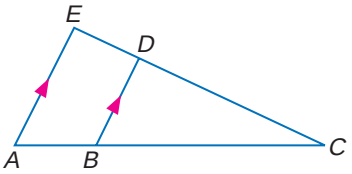
14.



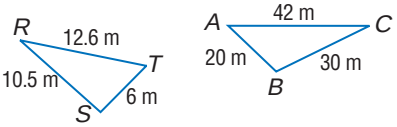
15.



16.

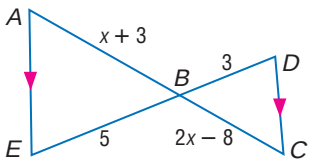


17.

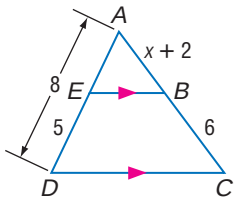


ALGEBRA **Identify the similar triangles, and find x and the measures of the indicated sides.**

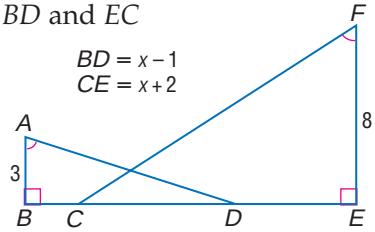
18. *AB* and *BC*



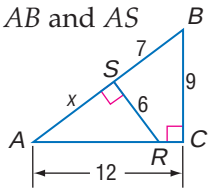
19. *AB* and *AC*



20. *BD* and *EC*



21. *AB* and *AS*



COORDINATE GEOMETRY **Triangles *ABC* and *TBS* have vertices *A*(−2, −8), *B*(4, 4), *C*(−2, 7), *T*(0, −4), and *S*(0, 6).**

22. Graph the triangles and prove that $\triangle ABC \sim \triangle TBS$.

23. Find the ratio of the perimeters of the two triangles.

Identify each statement as *true* or *false*. If false, state why.

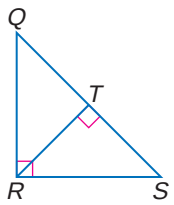
24. For every pair of similar triangles, there is only one correspondence of vertices that will give you correct angle correspondence and segment proportions.

25. If $\triangle ABC \sim \triangle EFG$ and $\triangle ABC \sim \triangle RST$, then $\triangle EFG \sim \triangle RST$.

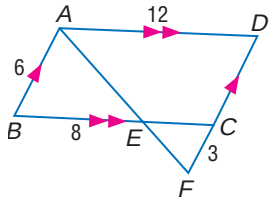
CONTENTS

Identify the similar triangles in each figure. Explain your answer.

26.

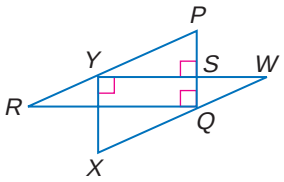


27.

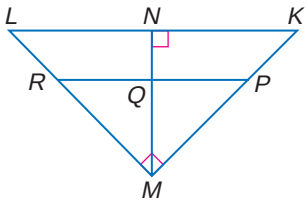


Use the given information to find each measure.

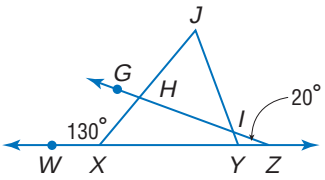
28. If $\overline{PR} \parallel \overline{WX}$, $WX = 10$, $XY = 6$, $WY = 8$, $RY = 5$, and $PS = 3$, find PY, SY, and PQ.



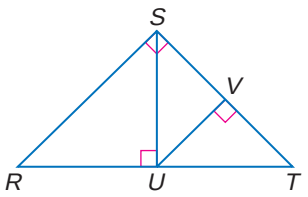
29. If $\overline{PR} \parallel \overline{KL}$, $KN = 9$, $LN = 16$, $PM = 2(KP)$, find KP, KM, MR, ML, MN, and PR.



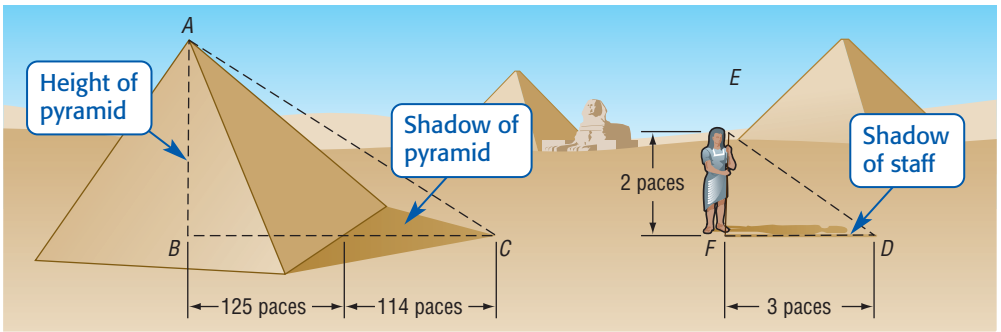
30. If $\frac{IJ}{XJ} = \frac{HJ}{YJ}$, $m\angle WXJ = 130$, and $m\angle WZG = 20$, find $m\angle YIZ$, $m\angle JHI$, $m\angle JIH$, $m\angle J$, and $m\angle JHG$.



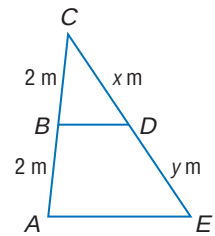
31. If $\angle RST$ is a right angle, $\overline{SU} \perp \overline{RT}$, $\overline{UV} \perp \overline{ST}$, and $m\angle RTS = 47$, find $m\angle TUV$, $m\angle R$, $m\angle RSU$, and $m\angle SUV$.



32. **HISTORY** The Greek mathematician Thales was the first to measure the height of a pyramid by using geometry. He showed that the ratio of a pyramid to a staff was equal to the ratio of one shadow to the other. If a pace is about 3 feet, approximately how tall was the pyramid at that time?



33. In the figure at the right, what relationship must be true of x and y for $\overline{BD}$ and $\overline{AE}$ to be parallel? Explain.



CONTENTS

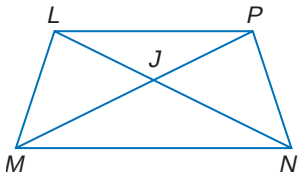
PROOF **For Exercises 34–38, write the type of proof specified.**

34. Write a two-column proof to show that if the measures of two sides of a triangle are proportional to the measures of two corresponding sides of another triangle and the included angles are congruent, then the triangles are similar. (Theorem 6.2)

35. a two-column proof

Given: $\overline{LP} \parallel \overline{MN}$

Prove: $\frac{LJ}{JN} = \frac{PJ}{JM}$

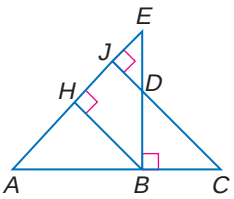


36. a paragraph proof

Given: $\overline{EB} \perp \overline{AC}$, $\overline{BH} \perp \overline{AE}$, $\overline{CJ} \perp \overline{AE}$

Prove: **a.** $\triangle ABH \sim \triangle DCB$

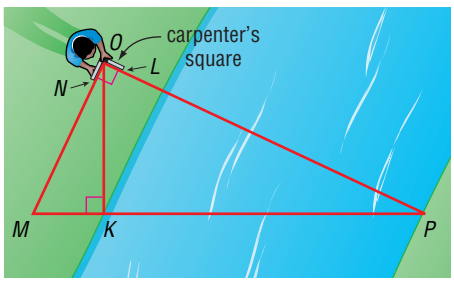
b. $\frac{BC}{BE} = \frac{BD}{BA}$



37. a two-column proof to show that if the measures of the legs of two right triangles are proportional, the triangles are similar

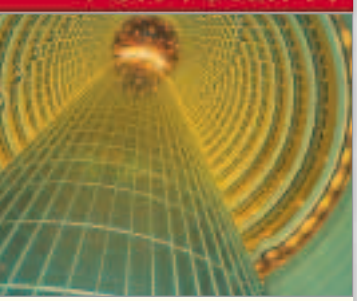
38. a two-column proof to prove that similarity of triangles is reflexive, symmetric, and transitive. (Theorem 6.3)

39. **SURVEYING** Mr. Glover uses a carpenter's square, an instrument used to draw right angles, to find the distance across a stream. The carpenter's square models right angle *NOL*. He puts the square on top of a pole that is high enough to sight along $\overline{OL}$ to point *P* across the river. Then he sights along $\overline{ON}$ to point *M*. If *MK* is 1.5 feet and *OK* = 4.5 feet, find the distance *KP* across the stream.



40. The lengths of three sides of triangle *ABC* are 6 centimeters, 4 centimeters, and 9 centimeters. Triangle *DEF* is similar to triangle *ABC*. The length of one of the sides of triangle *DEF* is 36 centimeters. What is the greatest perimeter possible for triangle *DEF*?

More About . . .



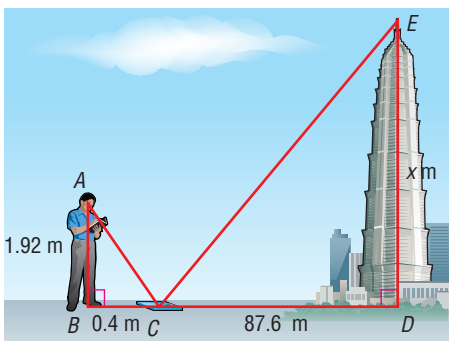
Towers

Completed in 1999, the Jin Mao Tower is 88 stories tall and sits on a 6-story podium, making it the tallest building in China. It is structurally engineered to tolerate typhoon winds and earthquakes.

Source: www.som.com

TOWERS **For Exercises 41 and 42, use the following information.**

To estimate the height of the Jin Mao Tower in Shanghai, a tourist sights the top of the tower in a mirror that is 87.6 meters from the tower. The mirror is on the ground and faces upward. The tourist is 0.4 meter from the mirror, and the distance from his eyes to the ground is about 1.92 meters.



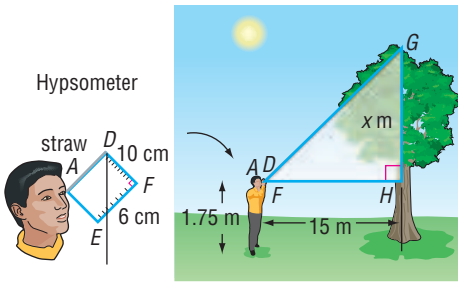
41. How tall is the tower?

42. Why is the mirror reflection a better way to indirectly measure the tower than by using shadows?

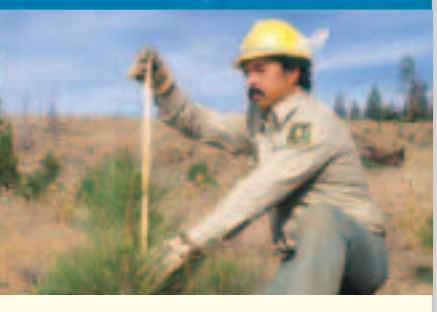
Macduff Everton/CORBIS

CONTENTS

43. FORESTRY A hypsometer as shown can be used to estimate the height of a tree. Bartolo looks through the straw to the top of the tree and obtains the readings given. Find the height of the tree.



Career Choices



Forester

Foresters plan and manage the growing, protection, and harvesting of trees. A 4-year degree in forestry stressing mathematics, science, computer science, communication skills, and technical forestry is required for a license in many states.

Online Research
For information about a career as a forester, visit www.geometryonline.com/careers

44. CRITICAL THINKING Suppose you know the height of a flagpole on the beach of the Chesapeake Bay and that it casts a shadow 4 feet long at 2:00 (EST). You also know the height of a flagpole on the shoreline of Lake Michigan whose shadow is hard to measure at 1:00 (CST). Since 2:00 (EST) = 1:00 (CST), you propose the following proportion of heights and lengths to find the length of the shadow of the Michigan flagpole. Explain whether this proportion will give an accurate measure.

$$\frac{\text{height of Chesapeake flagpole}}{\text{shadow of Chesapeake flagpole}} = \frac{\text{height of Michigan flagpole}}{\text{shadow of Michigan flagpole}}$$

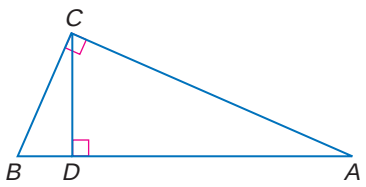
COORDINATE GEOMETRY **For Exercises 45 and 46, use the following information.**

The coordinates of $\triangle ABC$ are $A(-10, 6)$, $B(-2, 4)$, and $C(-4, -2)$. Point $D(6, 2)$ lies on $\overleftrightarrow{AB}$.

45. Graph $\triangle ABC$, point D, and draw $\overline{BD}$.

46. Where should a point E be located so that $\triangle ABC \sim \triangle ADE$?

47. CRITICAL THINKING The altitude $\overline{CD}$ from the right angle C in triangle ABC forms two triangles. Triangle ABC is similar to the two triangles formed, and the two triangles formed are similar to each other. Write three similarity statements about these triangles. Why are the triangles similar to each other?



48. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

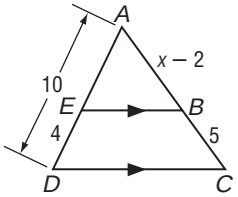
How do engineers use geometry?

Include the following in your answer:

- why engineers use triangles in construction, and
- why you think the pressure applied to the ground from the Eiffel Tower was so small.

Standardized Test Practice

49. If $\overline{EB} \parallel \overline{DC}$, find x.



(A) 9.5 (B) 5

(C) 4 (D) 2

50. ALGEBRA Solve $\frac{x+3}{6} = \frac{x}{x-2}$.

(A) 6 or 1 (B) 6 or −1

(C) 3 or 2 (D) −3 or 2

www.geometryonline.com/self_check_quiz

Lawrence Migdale/Stock Boston

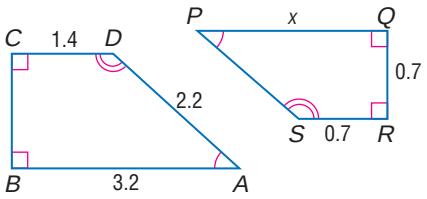
CONTENTS

Maintain Your Skills

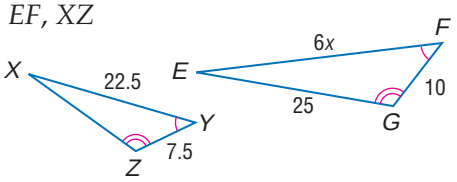
Mixed Review

Each pair of polygons is similar. Write a similarity statement, find x, the measures of the indicated sides, and the scale factor. *(Lesson 6-2)*

51. $\overline{BC}$, $\overline{PS}$



52. $\overline{EF}$, $\overline{XZ}$



Solve each proportion. *(Lesson 6-1)*

53. $\frac{1}{y} = \frac{3}{15}$ **54.** $\frac{6}{8} = \frac{7}{b}$ **55.** $\frac{20}{28} = \frac{m}{21}$ **56.** $\frac{16}{7} = \frac{9}{s}$

57. COORDINATE GEOMETRY $\triangle ABC$ has vertices $A(-3, -9)$, $B(5, 11)$, and $C(9, -1)$. $\overline{AT}$ is a median from A to $\overline{BC}$. Determine whether $\overline{AT}$ is an altitude. *(Lesson 5-1)*

58. ROLLER COASTERS The sign in front of the Electric Storm roller coaster states *ALL riders must be at least 54 inches tall to ride.* If Adam is 5 feet 8 inches tall, can he ride the Electric Storm? Which law of logic leads you to this conclusion? *(Lesson 2-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find the coordinates of the midpoint of the segment whose endpoints are given.** *(To review **finding coordinates of midpoints**, see Lesson 1-3.)*

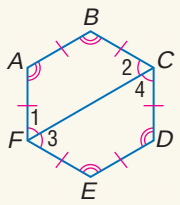
59. (2, 15), (9, 11) **60.** (−4, 4), (2, −12) **61.** (0, 8), (7, −13)

Practice Quiz 1

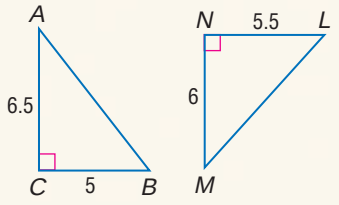
Lessons 6-1 through 6-3

Determine whether each pair of figures is similar. Justify your answer. *(Lesson 6-2)*

1.

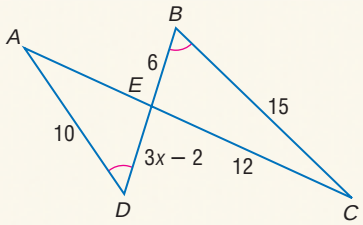


2.

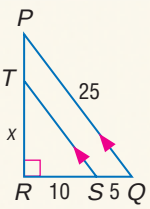


Identify the similar triangles. Find x and the measures of the indicated sides. *(Lesson 6-3)*

3. $\overline{AE}$, $\overline{DE}$



4. $\overline{PT}$, $\overline{ST}$



5. MAPS The scale on a map shows that 1.5 centimeters represents 100 miles. If the distance on the map from Atlanta, Georgia, to Los Angeles, California, is 29.2 centimeters, approximately how many miles apart are the two cities? *(Lesson 6-1)*

CONTENTS

6-4 Parallel Lines and Proportional Parts

What You'll Learn

- Use proportional parts of triangles.
- Divide a segment into parts.

How do city planners use geometry?

Street maps frequently have parallel and perpendicular lines. In Chicago, because of Lake Michigan, Lake Shore Drive runs at an angle between Oak Street and Ontario Street. City planners need to take this angle into account when determining dimensions of available land along Lake Shore Drive.

Vocabulary

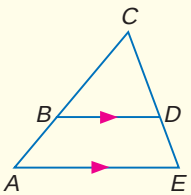
- midsegment

PROPORTIONAL PARTS OF TRIANGLES Nonparallel transversals that intersect parallel lines can be extended to form similar triangles. So the sides of the triangles are proportional.

Theorem 6.4

Triangle Proportionality Theorem If a line is parallel to one side of a triangle and intersects the other two sides in two distinct points, then it separates these sides into segments of proportional lengths.

Example: If $\overline{BD} \parallel \overline{AE}$, $\frac{BA}{CB} = \frac{DE}{CD}$.



Study Tip

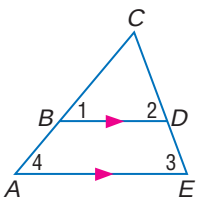
Overlapping Triangles

Trace two copies of $\triangle ACE$. Cut along $\overline{BD}$ to form $\triangle BCD$. Now $\triangle ACE$ and $\triangle BCD$ are no longer overlapping. Place the triangles side-by-side to compare corresponding angles and sides.

Proof Theorem 6.4

Given: $\overline{BD} \parallel \overline{AE}$

Prove: $\frac{BA}{CB} = \frac{DE}{CD}$



Paragraph Proof:

Since $\overline{BD} \parallel \overline{AE}$, $\angle 4 \cong \angle 1$ and $\angle 3 \cong \angle 2$ because they are corresponding angles. Then, by AA Similarity, $\triangle ACE \sim \triangle BCD$. From the definition of similar polygons, $\frac{CA}{CB} = \frac{CE}{CD}$. By the Segment Addition Postulate, $CA = BA + CB$ and $CE = DE + CD$. Substituting for CA and CE in the ratio, we get the following proportion.

$$\frac{BA + CB}{CB} = \frac{DE + CD}{CD}$$

$$\frac{BA}{CB} + \frac{CB}{CB} = \frac{DE}{CD} + \frac{CD}{CD} \quad \text{Rewrite as a sum.}$$

$$\frac{BA}{CB} + 1 = \frac{DE}{CD} + 1 \quad \frac{CB}{CB} = 1 \text{ and } \frac{CD}{CD} = 1$$

$$\frac{BA}{CB} = \frac{DE}{CD} \quad \text{Subtract 1 from each side.}$$

Example 1 Find the Length of a Side

In $\triangle EFG$, $\overline{HL} \parallel \overline{EF}$, $EH = 9$, $HG = 21$, and $FL = 6$. Find LG.

From the Triangle Proportionality Theorem, $\frac{EH}{HG} = \frac{FL}{LG}$.

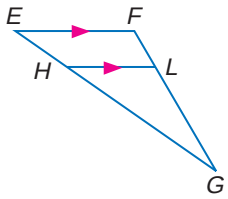
Substitute the known measures.

$$\frac{9}{21} = \frac{6}{LG}$$

$9(LG) = (21)6$ Cross products

$9(LG) = 126$ Multiply.

$LG = 14$ Divide each side by 9.



Study Tip

Using Fractions

You can also rewrite $\frac{9}{21}$ as $\frac{3}{7}$. Then use your knowledge of fractions to find the missing denominator.

$\times 2$

$\frac{3}{7} = \frac{6}{?}$

$\times 2$

The correct denominator is 14.

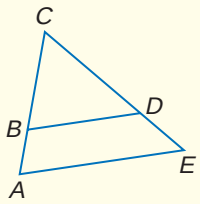
Proportional parts of a triangle can also be used to prove the converse of Theorem 6.4.

Theorem 6.5

Converse of the Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is parallel to the third side.

Example: If $\frac{BA}{CB} = \frac{DE}{CD}$, then $\overline{BD} \parallel \overline{AE}$.



You will prove Theorem 6.5 in Exercise 38.

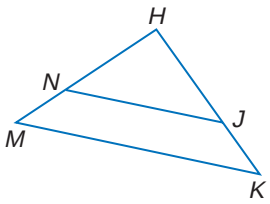
Example 2 Determine Parallel Lines

In $\triangle HKM$, $HM = 15$, $HN = 10$, and $\overline{HJ}$ is twice the length of $\overline{JK}$. Determine whether $\overline{NJ} \parallel \overline{MK}$. Explain.

$HM = HN + NM$ Segment Addition Postulate

$15 = 10 + NM$ $HM = 15$, $HN = 10$

$5 = NM$ Subtract 10 from each side.



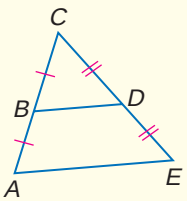
In order to show $\overline{NJ} \parallel \overline{MK}$, we must show that $\frac{HN}{NM} = \frac{HJ}{JK}$. $HN = 10$ and $NM = HM - HN$ or 5. So $\frac{HN}{NM} = \frac{10}{5}$ or 2. Let $JK = x$. Then $HJ = 2x$. So, $\frac{HJ}{JK} = \frac{2x}{x}$ or 2. Thus, $\frac{HN}{NM} = \frac{HJ}{JK} = 2$. Since the sides have proportional lengths, $\overline{NJ} \parallel \overline{MK}$.

A **midsegment** of a triangle is a segment whose endpoints are the midpoints of two sides of the triangle.

Theorem 6.6

Triangle Midsegment Theorem A midsegment of a triangle is parallel to one side of the triangle, and its length is one-half the length of that side.

Example: If B and D are midpoints of $\overline{AC}$ and $\overline{EC}$ respectively, $\overline{BD} \parallel \overline{AE}$ and $BD = \frac{1}{2}AE$.

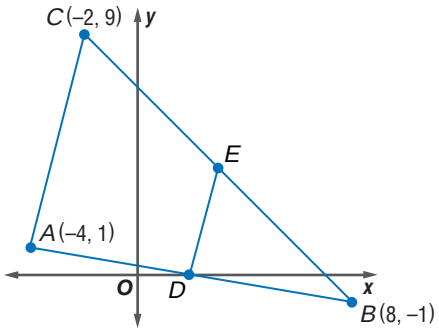


You will prove Theorem 6.6 in Exercise 39.

CONTENTS

Example 3 Midsegment of a Triangle

Triangle ABC has vertices $A(-4, 1)$, $B(8, -1)$, and $C(-2, 9)$. $\overline{DE}$ is a midsegment of $\triangle ABC$.



a. Find the coordinates of D and E.

Use the Midpoint Formula to find the midpoints of $\overline{AB}$ and $\overline{CB}$.

$$D\left(\frac{-4+8}{2}, \frac{1+(-1)}{2}\right) = D(2, 0)$$

$$E\left(\frac{-2+8}{2}, \frac{9+(-1)}{2}\right) = E(3, 4)$$

b. Verify that $\overline{AC}$ is parallel to $\overline{DE}$.

If the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

$$\text{slope of } \overline{AC} = \frac{9-1}{-2-(-4)} \text{ or } 4$$

$$\text{slope of } \overline{DE} = \frac{4-0}{3-2} \text{ or } 4$$

Because the slopes of $\overline{AC}$ and $\overline{DE}$ are equal, $\overline{AC} \parallel \overline{DE}$.

c. Verify that $DE = \frac{1}{2}AC$.

First, use the Distance Formula to find AC and DE.

$$AC = \sqrt{[-2-(-4)]^2 + (9-1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$DE = \sqrt{(3-2)^2 + (4-0)^2} = \sqrt{1+16} = \sqrt{17}$$

$$\frac{DE}{AC} = \frac{\sqrt{17}}{\sqrt{68}} = \sqrt{\frac{1}{4}} \text{ or } \frac{1}{2}$$

If $\frac{DE}{AC} = \frac{1}{2}$, then $DE = \frac{1}{2}AC$.

Study Tip

Look Back

To review the **Distance and Midpoint Formulas**, see Lesson 1-3.

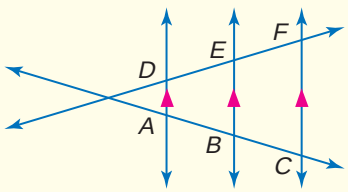
DIVIDE SEGMENTS PROPORTIONALLY

We have seen that parallel lines cut the sides of a triangle into proportional parts. Three or more parallel lines also separate transversals into proportional parts. If the ratio is 1, they separate the transversals into congruent parts.

Corollaries

6.1 If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.

Example: If $\overleftrightarrow{DA} \parallel \overleftrightarrow{EB} \parallel \overleftrightarrow{FC}$, then $\frac{AB}{BC} = \frac{DE}{EF}$, $\frac{AC}{DF} = \frac{BC}{EF}$, and $\frac{AC}{BC} = \frac{DF}{EF}$.



6.2 If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.

Example: If $\overline{AB} \cong \overline{BC}$, then $\overline{DE} \cong \overline{EF}$.

Study Tip

Three Parallel Lines

Corollary 6.1 is a special case of Theorem 6.4. In some drawings, the transversals are not shown to intersect. But, if extended, they will intersect and therefore, form triangles with each parallel line and the transversals.

Example 4 Proportional Segments

MAPS **Refer to the map showing part of Lake Shore Drive in Chicago at the beginning of the lesson. The streets from Oak Street to Ontario Street are all parallel to each other. The distance from Oak Street to Ontario along Michigan Avenue is about 3800 feet. The distance between the same two streets along Lake Shore Drive is about 4430 feet. If the distance from Delaware Place to Walton Street along Michigan Avenue is about 411 feet, what is the distance between those streets along Lake Shore Drive?**

Make a sketch of the streets in the problem. Notice that the streets form the bottom portion of a triangle that is cut by parallel lines. So you can use the Triangle Proportionality Theorem.

$$\frac{\text{Michigan Ave. Delaware to Walton}}{\text{Michigan Ave. Oak to Ontario}} = \frac{\text{Lake Shore Drive Delaware to Walton}}{\text{Lake Shore Drive Oak to Ontario}}$$ Triangle Proportionality Theorem

$$\frac{411}{3800} = \frac{x}{4430}$$ Substitution

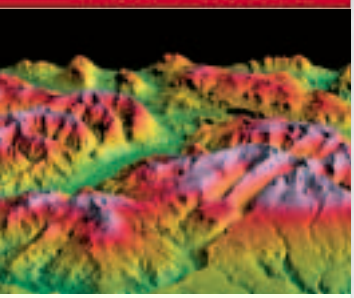
$$3800 \cdot x = 411(4430)$$ Cross products

$$3800x = 1{,}820{,}730$$ Multiply.

$$x = 479$$ Divide each side by 3800.

The distance from Delaware Place to Oak Street along Lake Shore Drive is about 479 feet.

More About . . .



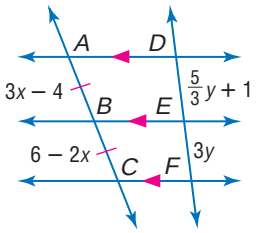
Maps

Modern map-making techniques involve using images taken from space to produce accurate representations on paper. In February 2000, the crew of the space shuttle *Endeavor* performed a radar topography mission, collecting a trillion radar images of 46 million square miles of Earth.

Source: www.nima.mil

Example 5 Congruent Segments

Find x and y.



To find x:

$AB = BC$ Given

$3x - 4 = 6 - 2x$ Substitution

$5x - 4 = 6$ Add $2x$ to each side.

$5x = 10$ Add 4 to each side.

$x = 2$ Divide each side by 5.

To find y:

$\overline{DE} \cong \overline{EF}$ Parallel lines that cut off congruent segments on one transversal cut off congruent segments on every transversal.

$DE \cong EF$ Definition of congruent segments

$3y = \frac{5}{3}y + 1$ Substitution

$9y = 5y + 3$ Multiply each side by 3 to eliminate the denominator.

$4y = 3$ Subtract $5y$ from each side.

$y = \frac{3}{4}$ Divide each side by 4.

JPL/NIMA/NASA

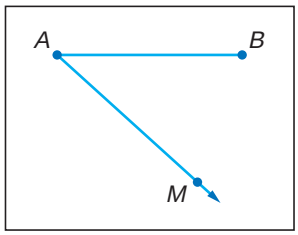
CONTENTS

It is possible to separate a segment into two congruent parts by constructing the perpendicular bisector of a segment. However, a segment cannot be separated into three congruent parts by constructing perpendicular bisectors. To do this, you must use parallel lines and the similarity theorems from this lesson.

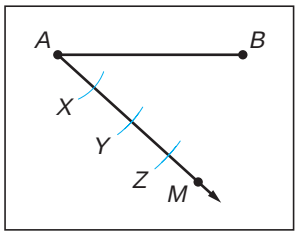
Construction

Trisect a Segment

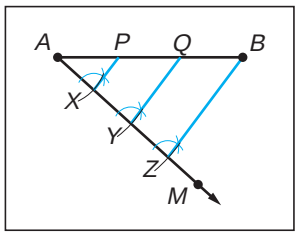
1. Draw $\overline{AB}$ to be trisected. Then draw $\overrightarrow{AM}$.



2. With the compass at A, mark off an arc that intersects $\overrightarrow{AM}$ at X. Use the same compass setting to construct $\overline{XY}$ and $\overline{YZ}$ congruent to $\overline{AX}$.



3. Draw $\overline{ZB}$. Then construct lines through Y and X that are parallel to $\overline{ZB}$. Label the intersection points on $\overline{AB}$ as P and Q.



Conclusion: Because parallel lines cut off congruent segments on transversals, $\overline{AP} \cong \overline{PQ} \cong \overline{QB}$.

Check for Understanding

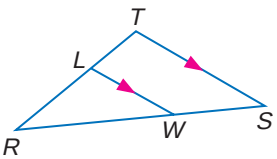
Concept Check

1. **Explain** how you would know if a line that intersects two sides of a triangle is parallel to the third side.
2. **OPEN ENDED** Draw two segments that are intersected by three lines so that the parts are proportional. Then draw a counterexample.
3. **Compare and contrast** Corollary 6.1 and Corollary 6.2.

Guided Practice

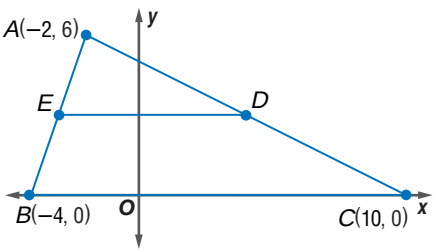
For Exercises 4 and 5, refer to $\triangle RST$.

4. If $RL = 5$, $RT = 9$, and $WS = 6$, find RW.
5. If $TR = 8$, $LR = 3$, and $RW = 6$, find WS.

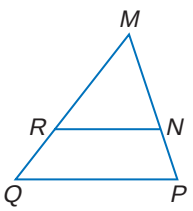


COORDINATE GEOMETRY **For Exercises 6–8, use the following information.**
Triangle ABC has vertices $A(-2, 6)$, $B(-4, 0)$, and $C(10, 0)$. $\overline{DE}$ is a midsegment.

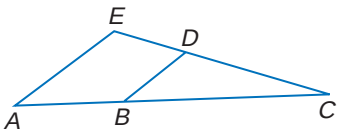
6. Find the coordinates of D and E.
7. Verify that $\overline{DE}$ is parallel to $\overline{BC}$.
8. Verify that $DE = \frac{1}{2}BC$.



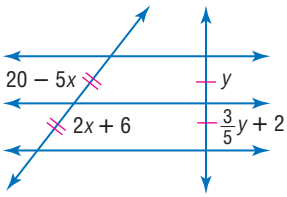
9. In $\triangle MQP$, $MP = 25$, $MN = 9$, $MR = 4.5$, and $MQ = 12.5$. Determine whether $\overline{RN} \parallel \overline{QP}$. Justify your answer.



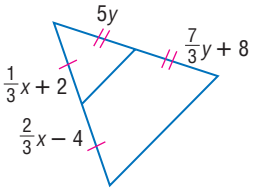
10. In $\triangle ACE$, $ED = 8$, $DC = 20$, $BC = 25$, and $AB = 12$. Determine whether $\overline{DB} \parallel \overline{AE}$.



11. Find x and y.

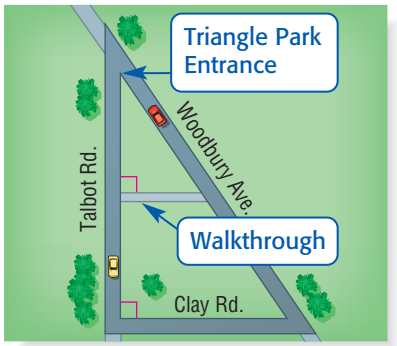


12. Find x and y.



Application

13. MAPS The distance along Talbot Road from the Triangle Park entrance to the Walkthrough is 880 yards. The distance along Talbot Road from the Walkthrough to Clay Road is 1408 yards. The distance along Woodbury Avenue from the Walkthrough to Clay Road is 1760 yards. If the Walkthrough is parallel to Clay Road, find the distance from the entrance to the Walkthrough along Woodbury.



Practice and Apply

Homework Help

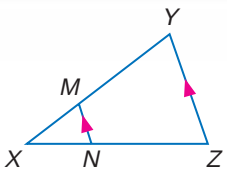
For Exercises	See Examples
14–19	1
20–26	2
27, 28	3
35–37, 43	4
33, 34	5

Extra Practice
See page 765.

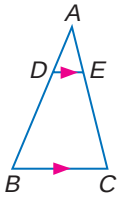
For Exercises 14 and 15, refer to $\triangle XYZ$.

14. If $XM = 4$, $XN = 6$, and $NZ = 9$, find XY.

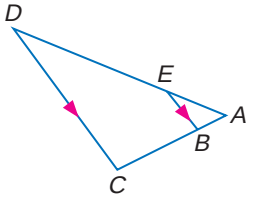
15. If $XN = t - 2$, $NZ = t + 1$, $XM = 2$, and $XY = 10$, solve for t.



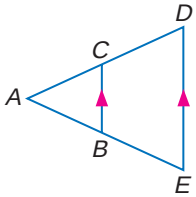
16. If $DB = 24$, $AE = 3$, and $EC = 18$, find AD.



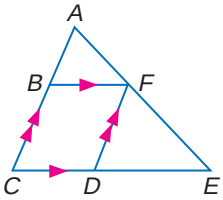
17. Find x and ED if $AE = 3$, $AB = 2$, $BC = 6$, and $ED = 2x - 3$.



18. Find x, AC, and CD if $AC = x - 3$, $BE = 20$, $AB = 16$, and $CD = x + 5$.



19. Find BC, FE, CD, and DE if $AB = 6$, $AF = 8$, $BC = x$, $CD = y$, $DE = 2y - 3$, and $FE = x + \frac{10}{3}$.



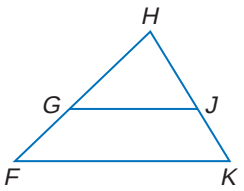
CONTENTS

Find x so that $\overline{GJ} \parallel \overline{FK}$.

20. $GF = 12$, $HG = 6$, $HJ = 8$, $JK = x - 4$

21. $HJ = x - 5$, $JK = 15$, $FG = 18$, $HG = x - 4$

22. $GH = x + 3.5$, $HJ = x - 8.5$, $FH = 21$, $HK = 7$



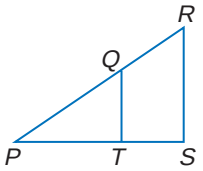
Determine whether $\overline{QT} \parallel \overline{RS}$. Justify your answer.

23. $PR = 30$, $PQ = 9$, $PT = 12$, and $PS = 18$

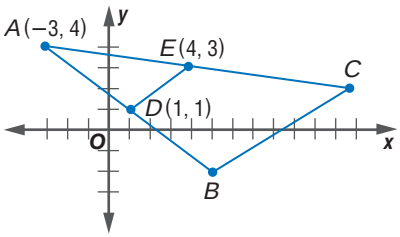
24. $QR = 22$, $RP = 65$, and SP is 3 times TS.

25. $TS = 8.6$, $PS = 12.9$, and PQ is half RQ.

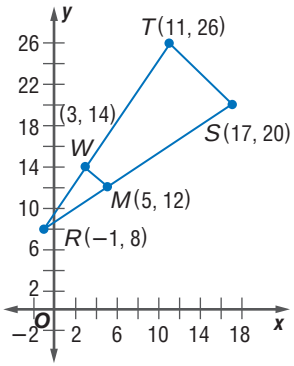
26. $PQ = 34.88$, $RQ = 18.32$, $PS = 33.25$, and $TS = 11.45$



27. Find the length of $\overline{BC}$ if $\overline{BC} \parallel \overline{DE}$ and $\overline{DE}$ is a midsegment of $\triangle ABC$.



28. Show that $\overline{WM} \parallel \overline{TS}$ and determine whether $\overline{WM}$ is a midsegment.

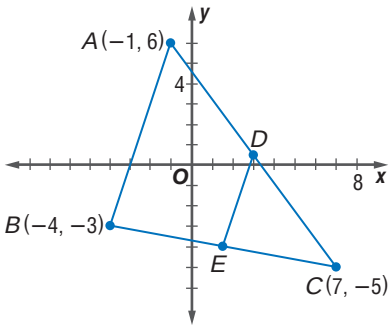


COORDINATE GEOMETRY **For Exercises 29 and 30, use the following information.**

Triangle ABC has vertices $A(-1, 6)$, $B(-4, -3)$, and $C(7, -5)$. $\overline{DE}$ is a midsegment.

29. Verify that $\overline{DE}$ is parallel to $\overline{AB}$.

30. Verify that $DE = \frac{1}{2}AB$.

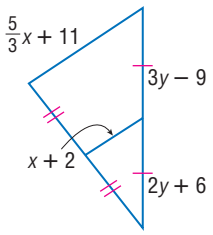


31. **COORDINATE GEOMETRY** Given $A(2, 12)$ and $B(5, 0)$, find the coordinates of P such that P separates $\overline{AB}$ into two parts with a ratio of 2 to 1.

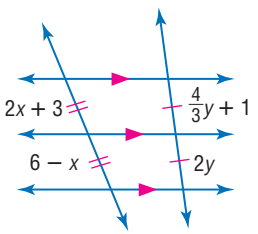
32. **COORDINATE GEOMETRY** In $\triangle LMN$, $\overline{PR}$ divides $\overline{NL}$ and $\overline{MN}$ proportionally. If the vertices are $N(8, 20)$, $P(11, 16)$, and $R(3, 8)$ and $\frac{LP}{PN} = \frac{2}{1}$, find the coordinates of L and M.

ALGEBRA **Find x and y.**

33.



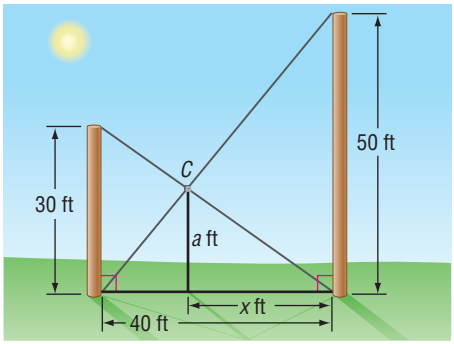
34.



CONTENTS

CONSTRUCTION **For Exercises 35–37, use the following information and drawing.**

Two poles, 30 feet and 50 feet tall, are 40 feet apart and perpendicular to the ground. The poles are supported by wires attached from the top of each pole to the bottom of the other, as in the figure. A coupling is placed at C where the two wires cross.



35. Find x, the distance from C to the taller pole.

36. How high above the ground is the coupling?

37. How far down the wire from the smaller pole is the coupling?

PROOF **Write a two-column proof of each theorem.**

38. Theorem 6.5

39. Theorem 6.6

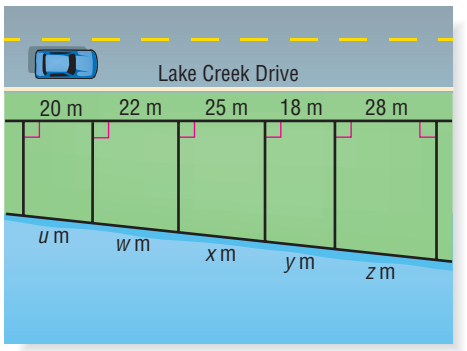
CONSTRUCTION **Construct each segment as directed.**

40. a segment 8 centimeters long, separated into three congruent segments

41. a segment separated into four congruent segments

42. a segment separated into two segments in which their lengths have a ratio of 1 to 4

43. REAL ESTATE In Lake Creek, the lots on which houses are to be built are laid out as shown. What is the lake frontage for each of the five lots if the total frontage is 135.6 meters?



44. CRITICAL THINKING Copy the figure that accompanies Corollary 6.1 on page 309. Draw $\overline{DC}$. Let G be the intersection point of $\overline{DC}$ and $\overline{BE}$. Using that segment, explain how you could prove $\frac{AB}{BC} = \frac{DE}{EF}$.

45. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do city planners use geometry?

Include the following in your answer:

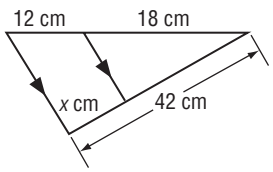
- why maps are important to city planners, and
- what geometry facts a city planner needs to know to explain why the block between Chestnut and Pearson is longer on Lake Shore Drive than on Michigan Avenue.

Standardized Test Practice

46. Find x.

Ⓐ 16 Ⓑ 16.8

Ⓒ 24 Ⓓ 28.4



47. GRID IN The average of a and b is 18, and the ratio of a to b is 5 to 4. What is the value of $a - b$?

CONTENTS

Extending the Lesson

48. **MIDPOINTS IN POLYGONS** **Draw any quadrilateral $ABCD$ on a coordinate plane. Points E, F, G, and H are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$, respectively.**

a. Connect the midpoints to form quadrilateral $EFGH$. Describe what you know about the sides of quadrilateral $EFGH$.

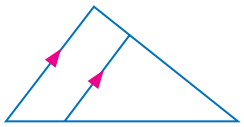
b. Will the same reasoning work with five-sided polygons? Explain why or why not.

Maintain Your Skills

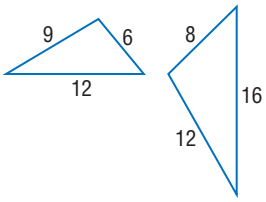
Mixed Review

Determine whether each pair of triangles is similar. Justify your answer. *(Lesson 6-3)*

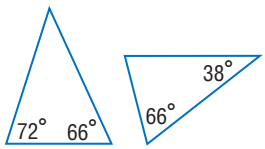
49.



50.

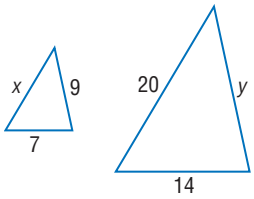


51.

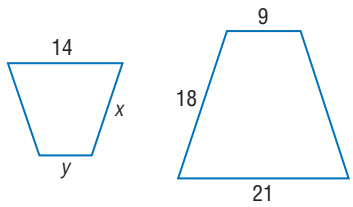


Each pair of polygons is similar. Find x and y. *(Lesson 6-2)*

52.



53.



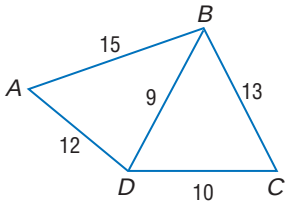
Determine the relationship between the measures of the given angles. *(Lesson 5-2)*

54. $\angle ADB$, $\angle ABD$

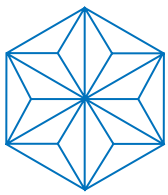
55. $\angle ABD$, $\angle BAD$

56. $\angle BCD$, $\angle CDB$

57. $\angle CBD$, $\angle BCD$



ARCHITECTURE **For Exercises 58 and 59, use the following information.**
The geodesic dome was developed by Buckminster Fuller in the 1940s as an energy-efficient building. The figure at the right shows the basic structure of one geodesic dome. *(Lesson 4-1)*



58. How many equilateral triangles are in the figure?

59. How many obtuse triangles are in the figure?

Determine the truth value of the following statement for each set of conditions.
If you have a fever, then you are sick. *(Lesson 2-3)*

60. You do not have a fever, and you are sick.

61. You have a fever, and you are not sick.

62. You do not have a fever, and you are not sick.

63. You have a fever, and you are sick.

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write all the pairs of corresponding parts for each pair of congruent triangles.** *(To review **corresponding congruent parts**, see Lesson 4-3.)*

64. $\triangle ABC \cong \triangle DEF$ **65.** $\triangle RST \cong \triangle XYZ$ **66.** $\triangle PQR \cong \triangle KLM$

6-5 Parts of Similar Triangles

What You'll Learn

- Recognize and use proportional relationships of corresponding perimeters of similar triangles.
- Recognize and use proportional relationships of corresponding angle bisectors, altitudes, and medians of similar triangles.

How is geometry related to photography?

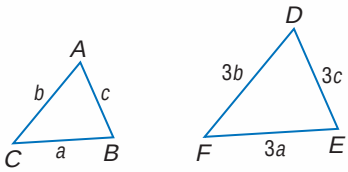
The camera lens was 6.16 meters from this Dale Chihuly glass sculpture when this photograph was taken. The image on the film is 35 millimeters tall. Similar triangles enable us to find the height of the actual sculpture.

More About...

Dale Chihuly

Dale Chihuly (1941–), born in Tacoma, Washington, is widely recognized as one of the greatest glass artists in the world. His sculptures are made of hundreds of pieces of hand-blown glass that are assembled to resemble patterns in nature.

PERIMETERS Triangle ABC is similar to $\triangle DEF$ with a scale factor of 1:3. You can use variables and the scale factor to compare their perimeters. Let the measures of the sides of $\triangle ABC$ be a, b, and c. The measures of the corresponding sides of $\triangle DEF$ would be $3a$, $3b$, and $3c$.



$$\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{a + b + c}{3a + 3b + 3c}$$
$$= \frac{1(a + b + c)}{3(a + b + c)} \text{ or } \frac{1}{3}$$

The perimeters are in the same proportion as the side measures of the two similar figures. This suggests Theorem 6.7, the Proportional Perimeters Theorem.

Theorem 6.7

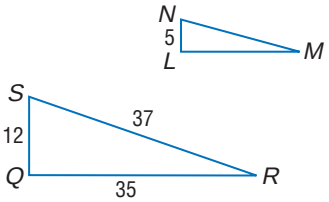
Proportional Perimeters Theorem If two triangles are similar, then the perimeters are proportional to the measures of corresponding sides.

You will prove Theorem 6.7 in Exercise 8.

Example 1 Perimeters of Similar Triangles

If $\triangle LMN \sim \triangle QRS$, $QR = 35$, $RS = 37$, $SQ = 12$, and $NL = 5$, find the perimeter of $\triangle LMN$.

Let x represent the perimeter of $\triangle LMN$. The perimeter of $\triangle QRS = 35 + 37 + 12$ or 84.



$\frac{NL}{SQ} = \frac{\text{perimeter of } \triangle LMN}{\text{perimeter of } \triangle QRS}$ Proportional Perimeter Theorem

$\frac{5}{12} = \frac{x}{84}$ Substitution

$12x = 420$ Cross products

$x = 35$ Divide each side by 12.

The perimeter of $\triangle LMN$ is 35 units.

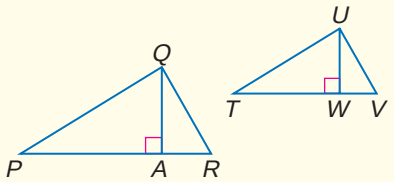
(l)Kelly-Mooney Photography/CORBIS, (r)Pierre Burnaugh/PhotoEdit
CONTENTS

SPECIAL SEGMENTS OF SIMILAR TRIANGLES Think about a triangle drawn on a piece of paper being placed on a copy machine and either enlarged or reduced. The copy is similar to the original triangle. Now suppose you drew in special segments of a triangle, such as the altitudes, medians, or angle bisectors, on the original. When you enlarge or reduce that original triangle, all of those segments are enlarged or reduced at the same rate. This conjecture is formally stated in Theorems 6.8, 6.9, and 6.10.

Theorems — Special Segments of Similar Triangles

6.8 If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides.

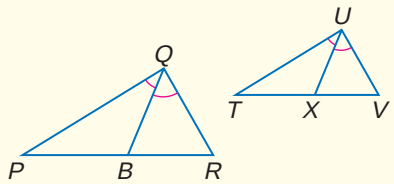
Abbreviation: ~ △s *have corr. altitudes proportional to the corr. sides.*



$$\frac{QA}{UW} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

6.9 If two triangles are similar, then the measures of the corresponding angle bisectors of the triangles are proportional to the measures of the corresponding sides.

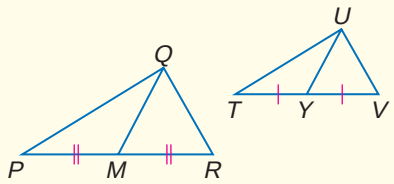
Abbreviation: ~ △s *have corr. ∠ bisectors proportional to the corr. sides.*



$$\frac{QB}{UX} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

6.10 If two triangles are similar, then the measures of the corresponding medians are proportional to the measures of the corresponding sides.

Abbreviation: ~ △s *have corr. medians proportional to the corr. sides.*



$$\frac{QM}{UY} = \frac{PR}{TV} = \frac{QR}{UV} = \frac{PQ}{TU}$$

You will prove Theorems 6.8 and 6.10 in Exercises 30 and 31, respectively.

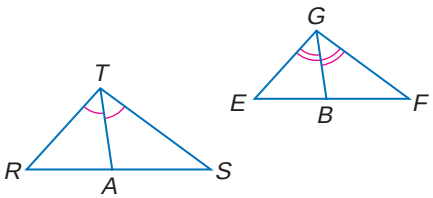
Example 2 Write a Proof

Write a paragraph proof of Theorem 6.9.

Since the corresponding angles to be bisected are chosen at random, we need not prove this for every pair of bisectors.

Given: $\triangle RTS \sim \triangle EGF$
$\overline{TA}$ and $\overline{GB}$ are angle bisectors.

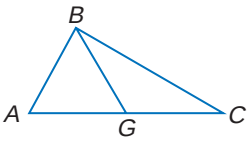
Prove: $\frac{TA}{GB} = \frac{RT}{EG}$

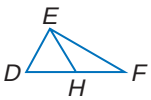


Paragraph Proof: Because corresponding angles of similar triangles are congruent, $\angle R \cong \angle E$ and $\angle RTS \cong \angle EGF$. Since $\angle RTS$ and $\angle EGF$ are bisected, we know that $\frac{1}{2}m\angle RTS = \frac{1}{2}m\angle EGF$ or $m\angle RTA = m\angle EGB$. This makes $\angle RTF \cong \angle EGB$ and $\triangle RTF \sim \triangle EGB$ by AA Similarity. Thus, $\frac{TA}{GB} = \frac{RT}{EG}$.

Example 3 Medians of Similar Triangles

In the figure, $\triangle ABC \sim \triangle DEF$. $\overline{BG}$ is a median of $\triangle ABC$, and $\overline{EH}$ is a median of $\triangle DEF$. Find EH if $BC = 30$, $BG = 15$, and $EF = 15$.





Let x represent EH.

$\frac{BG}{EH} = \frac{BC}{EF}$ Write a proportion.

$\frac{15}{x} = \frac{30}{15}$ $BG = 15$, $EH = x$, $BC = 30$, and $EF = 15$

$30x = 225$ Cross products

$x = 7.5$ Divide each side by 30.

Thus, $EH = 7.5$.

More About...

Photography

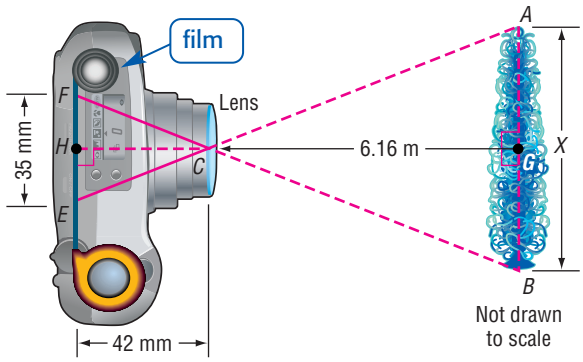
The first consumer-oriented digital cameras were produced for sale in 1994 with a 640 × 480 pixel resolution. In 2002, a 3.3-megapixel camera could take a picture with 2048 × 1536 pixel resolution, which is a sharper picture than most computer monitors can display.

Source: www.howstuffworks.com

The theorems about the relationships of special segments in similar triangles can be used to solve real-life problems.

Example 4 Solve Problems with Similar Triangles

PHOTOGRAPHY **Refer to the application at the beginning of the lesson. The drawing below illustrates the position of the camera and the distance from the lens of the camera to the film. Find the height of the sculpture.**



$\triangle ABC$ and $\triangle EFC$ are similar. The distance from the lens to the film in the camera is $CH = 42$ mm. $\overline{CG}$ and $\overline{CH}$ are altitudes of $\triangle ABC$ and $\triangle EFC$, respectively. If two triangles are similar, then the measures of the corresponding altitudes are proportional to the measures of the corresponding sides. This leads to the proportion $\frac{AB}{EF} = \frac{GC}{HC}$.

$\frac{AB}{EF} = \frac{GC}{HC}$ Write the proportion.

$\frac{x \text{ m}}{35 \text{ mm}} = \frac{6.16 \text{ m}}{42 \text{ mm}}$ $AB = x$ m, $EF = 35$ m, $GC = 6.16$ m, $HC = 42$ mm

$x \cdot 42 = 35(6.16)$ Cross products

$42x = 215.6$ Simplify.

$x \approx 5.13$ Divide each side by 42.

The sculpture is about 5.13 meters tall.

Beth A. Keiser/AP/Wide World Photos

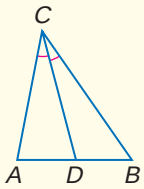
CONTENTS

An angle bisector also divides the side of the triangle opposite the angle proportionally.

Theorem 6.11

Angle Bisector Theorem An angle bisector in a triangle separates the opposite side into segments that have the same ratio as the other two sides.

Example: $\frac{AD}{DB} = \frac{AC}{BC}$ ← segments with vertex A ← segments with vertex B

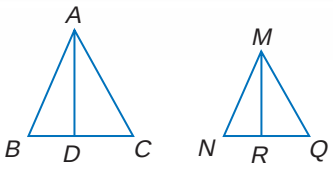


You will prove this theorem in Exercise 32.

Check for Understanding

Concept Check

1. Explain what must be true about $\triangle ABC$ and $\triangle MNQ$ before you can conclude that $\frac{AD}{MR} = \frac{BA}{NM}$.



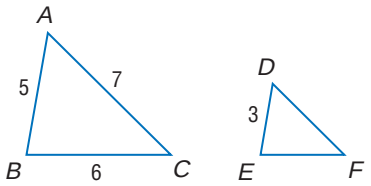
2. OPEN ENDED The perimeter of one triangle is 24 centimeters, and the perimeter of a second triangle is 36 centimeters. If the length of one side of the smaller triangle is 6, find possible lengths of the other sides of the triangles so that they are similar.

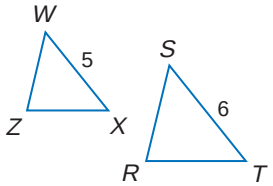
Guided Practice

Find the perimeter of the given triangle.

3. $\triangle DEF$, if $\triangle ABC \sim \triangle DEF$, $AB = 5$, $BC = 6$, $AC = 7$, and $DE = 3$

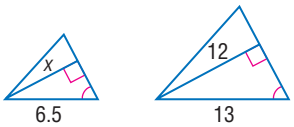
4. $\triangle WZX$, if $\triangle WZX \sim \triangle SRT$, $ST = 6$, $WX = 5$, and the perimeter of $\triangle SRT = 15$



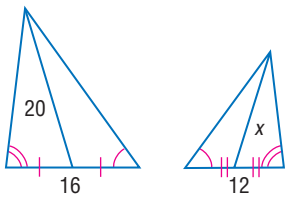


Find x.

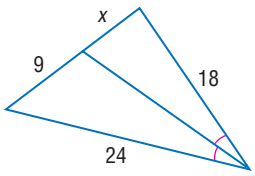
5.



6.



7.

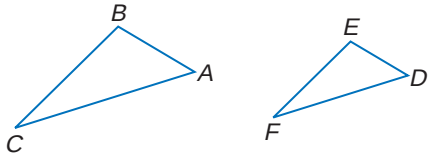


8. PROOF Write a paragraph proof of Theorem 6.7.

Given: $\triangle ABC \sim \triangle DEF$

$\frac{AB}{DE} = \frac{m}{n}$

Prove: $\frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF} = \frac{m}{n}$



Application

9. PHOTOGRAPHY The distance from the film to the lens in a camera is 10 centimeters. The film image is 5 centimeters high. Tamika is 165 centimeters tall. How far should she be from the camera in order for the photographer to take a full-length picture?

Practice and Apply

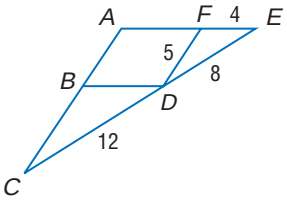
Homework Help

For Exercises	See Examples
10–15	1
16, 17, 28	4
18–27	3
30–37	2

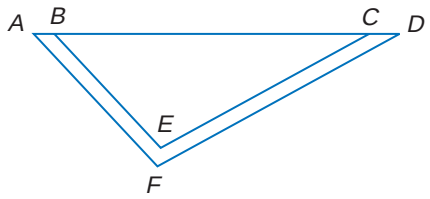
Extra Practice
See page 766.

Find the perimeter of the given triangle.

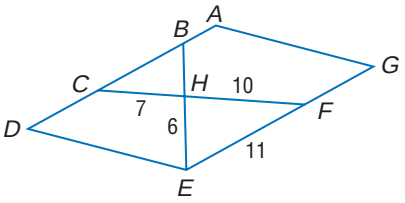
10. $\triangle BCD$, if $\triangle BCD \sim \triangle FDE$, $CD = 12$, $FD = 5$, $FE = 4$, and $DE = 8$



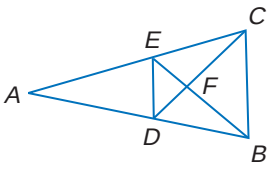
11. $\triangle ADF$, if $\triangle ADF \sim \triangle BCE$, $BC = 24$, $EB = 12$, $CE = 18$, and $DF = 21$



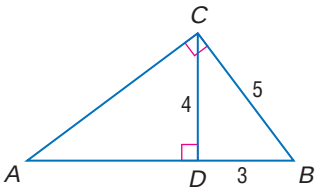
12. $\triangle CBH$, if $\triangle CBH \sim \triangle FEH$, $ADEG$ is a parallelogram, $CH = 7$, $FH = 10$, $FE = 11$, and $EH = 6$



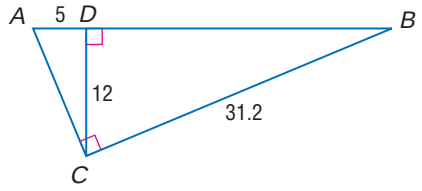
13. $\triangle DEF$, if $\triangle DEF \sim \triangle CBF$, perimeter of $\triangle CBF = 27$, $DF = 6$, and $FC = 8$



14. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $CD = 4$, $DB = 3$, and $CB = 5$

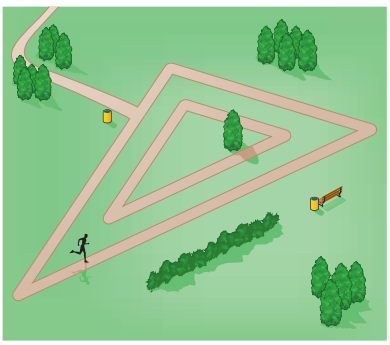


15. $\triangle ABC$, if $\triangle ABC \sim \triangle CBD$, $AD = 5$, $CD = 12$, and $BC = 31.2$

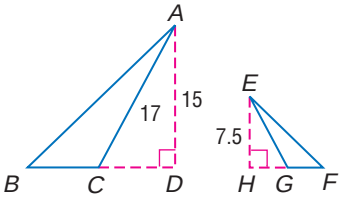


16. DESIGN Rosario wants to enlarge the dimensions of an 18-centimeter by 24-centimeter picture by 30%. She plans to line the inside edge of the frame with blue cord. The store only had 110 centimeters of blue cord in stock. Will this be enough to fit on the inside edge of the frame? Explain.

17. PHYSICAL FITNESS A park has two similar triangular jogging paths as shown. The dimensions of the inner path are 300 meters, 350 meters, and 550 meters. The shortest side of the outer path is 600 meters. Will a jogger on the inner path run half as far as one on the outer path? Explain.

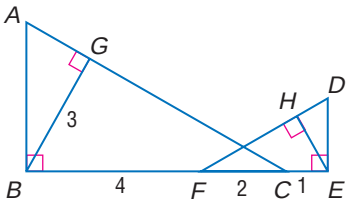


18. Find EG if $\triangle ACB \sim \triangle EGF$, $\overline{AD}$ is an altitude of $\triangle ACB$, $\overline{EH}$ is an altitude of $\triangle EGF$, $AC = 17$, $AD = 15$, and $EH = 7.5$.

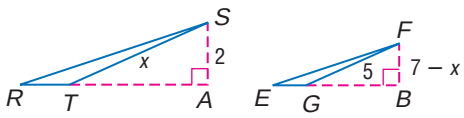


CONTENTS

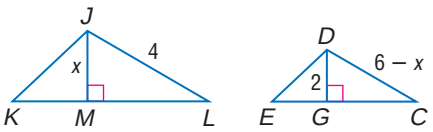
19. Find EH if $\triangle ABC \sim \triangle DEF$, $\overline{BG}$ is an altitude of $\triangle ABC$, $\overline{EH}$ is an altitude of $\triangle DEF$, $BG = 3$, $BF = 4$, $FC = 2$, and $CE = 1$.



20. Find FB if $\overline{SA}$ and $\overline{FB}$ are altitudes and $\triangle RST \sim \triangle EFG$.

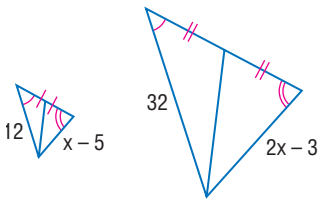


21. Find DC if $\overline{DG}$ and $\overline{JM}$ are altitudes and $\triangle KJL \sim \triangle EDC$.

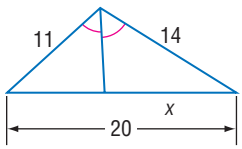


Find x.

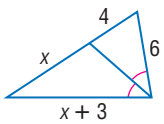
22.



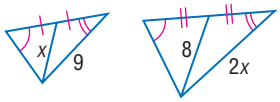
23.



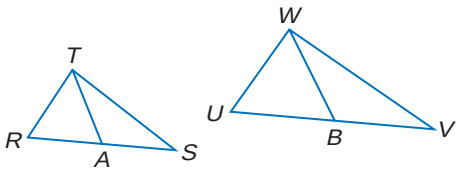
24.



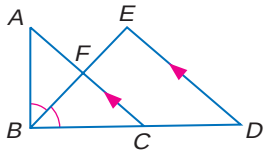
25.



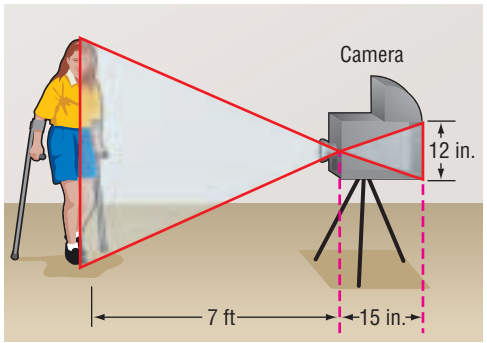
26. Find UB if $\triangle RST \sim \triangle UVW$, $\overline{TA}$ and $\overline{WB}$ are medians, $TA = 8$, $RA = 3$, $WB = 3x - 6$, and $UB = x + 2$.



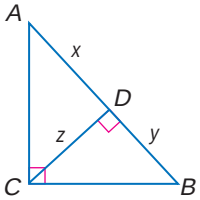
27. Find CF and BD if $\overline{BF}$ bisects $\angle ABC$ and $\overline{AC} \parallel \overline{ED}$, $BA = 6$, $BC = 7.5$, $AC = 9$, and $DE = 9$.



28. PHOTOGRAPHY One of the first cameras invented was called a *camera obscura*. Light entered an opening in the front, and an image was reflected in the back of the camera, upside down, forming similar triangles. If the image of the person on the back of the camera is 12 inches, the distance from the opening to the person is 7 feet, and the camera itself is 15 inches long, how tall is the person being photographed?



29. CRITICAL THINKING $\overline{CD}$ is an altitude to the hypotenuse $\overline{AB}$. Make a conjecture about x, y, and z. Justify your reasoning.



CONTENTS

PROOF **Write the indicated type of proof.**

30. a paragraph proof of Theorem 6.8

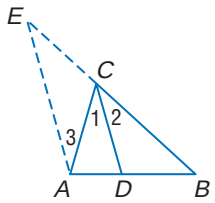
31. a two-column proof of Theorem 6.10

32. a two-column proof of the Angle Bisector Theorem (Theorem 6.11)

Given: $\overline{CD}$ bisects $\angle ACB$

By construction, $\overline{AE} \parallel \overline{CD}$.

Prove: $\frac{AD}{AC} = \frac{BD}{BC}$



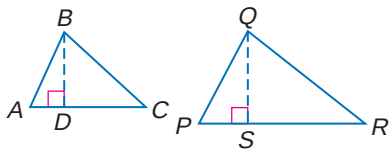
33. a paragraph proof

Given: $\triangle ABC \sim \triangle PQR$

$\overline{BD}$ is an altitude of $\triangle ABC$.

$\overline{QS}$ is an altitude of $\triangle PQR$.

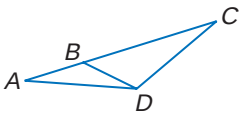
Prove: $\frac{QP}{BA} = \frac{QS}{BD}$



34. a flow proof

Given: $\angle C \cong \angle BDA$

Prove: $\frac{AC}{DA} = \frac{AD}{BA}$

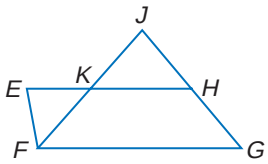


35. a two-column proof

Given: $\overline{JF}$ bisects $\angle EFG$.

$\overline{EH} \parallel \overline{FG}$, $\overline{EF} \parallel \overline{HG}$

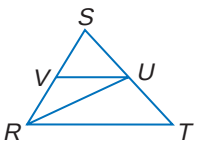
Prove: $\frac{EK}{KF} = \frac{GJ}{JF}$



36. a two-column proof

Given: $\overline{RU}$ bisects $\angle SRT$; $\overline{VU} \parallel \overline{RT}$.

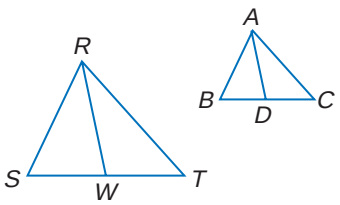
Prove: $\frac{SV}{VR} = \frac{SR}{RT}$



37. a flow proof

Given: $\triangle RST \sim \triangle ABC$; W and D are midpoints of $\overline{TS}$ and $\overline{CB}$.

Prove: $\triangle RWS \sim \triangle ADB$



38. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is geometry related to photography?

Include the following in your answer:

- a sketch of how a camera works showing the image and the film, and
- why the two isosceles triangles are similar.

Standardized Test Practice

39. **GRID IN** Triangle ABC is similar to $\triangle DEF$. If $AC = 10.5$, $AB = 6.5$, and $DE = 8$, find DF.

40. **ALGEBRA** The sum of three numbers is 180. Two of the numbers are the same, and each of them is one-third of the greatest number. What is the least number?

(A) 30 (B) 36 (C) 45 (D) 72

CONTENTS

Maintain Your Skills

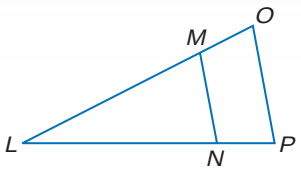
Mixed Review

Determine whether $\overline{MN} \parallel \overline{OP}$. Justify your answer. *(Lesson 6-4)*

41. $LM = 7, LN = 9, LO = 14, LP = 16$

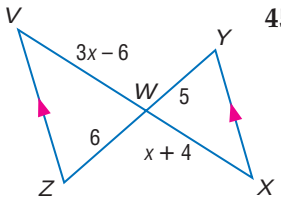
42. $LM = 6, MN = 4, LO = 9, OP = 6$

43. $LN = 12, NP = 4, LM = 15, MO = 5$

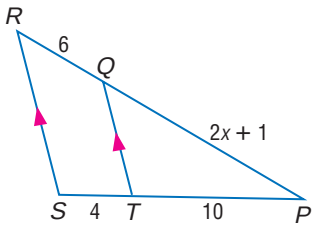


Identify the similar triangles. Find x and the measure(s) of the indicated side(s). *(Lesson 6-3)*

44. VW and WX



45. PQ



Write an equation in slope-intercept form for the line that satisfies the given conditions. *(Lesson 3-4)*

46. x-intercept is 3, y-intercept is -3

47. $m = 2$, contains $(-1, -1)$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Name the next two numbers in each pattern.**
*(To review **patterns**, see Lesson 2-1.)*

48. 5, 12, 19, 26, 33, …

49. 10, 20, 40, 80, 160, …

50. 0, 5, 4, 9, 8, 13, …

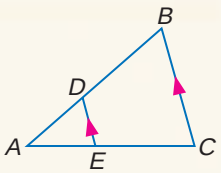
Practice Quiz 2

Lessons 6-4 and 6-5

Refer to $\triangle ABC$. *(Lesson 6-4)*

1. If $AD = 8$, $AE = 12$, and $EC = 18$, find AB.

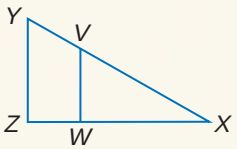
2. If $AE = m - 2$, $EC = m + 4$, $AD = 4$, and $AB = 20$, find m.



Determine whether $\overline{YZ} \parallel \overline{VW}$. Justify your answer. *(Lesson 6-4)*

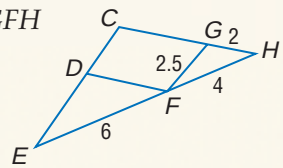
3. $XY = 30$, $XV = 9$, $XW = 12$, and $XZ = 18$

4. $XV = 34.88$, $VY = 18.32$, $XZ = 33.25$, and $WZ = 11.45$

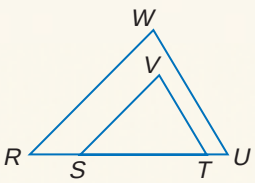


Find the perimeter of the given triangle. *(Lesson 6-5)*

5. $\triangle DEF$ if $\triangle DEF \sim \triangle GFH$

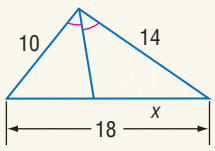


6. $\triangle RUW$ if $\triangle RUW \sim \triangle STV$, $ST = 24$, $VS = 12$, $VT = 18$, and $UW = 21$

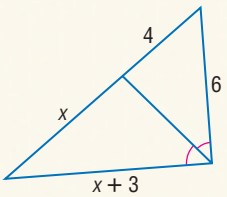


Find x. *(Lesson 6-5)*

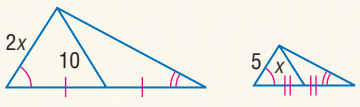
7.



8.



9.



10. LANDSCAPING Paulo is designing two gardens shaped like similar triangles. One garden has a perimeter of 53.5 feet, and the longest side is 25 feet. He wants the second garden to have a perimeter of 32.1 feet. Find the length of the longest side of this garden. *(Lesson 6-5)*

Geometry Activity

A Preview of Lesson 6-6

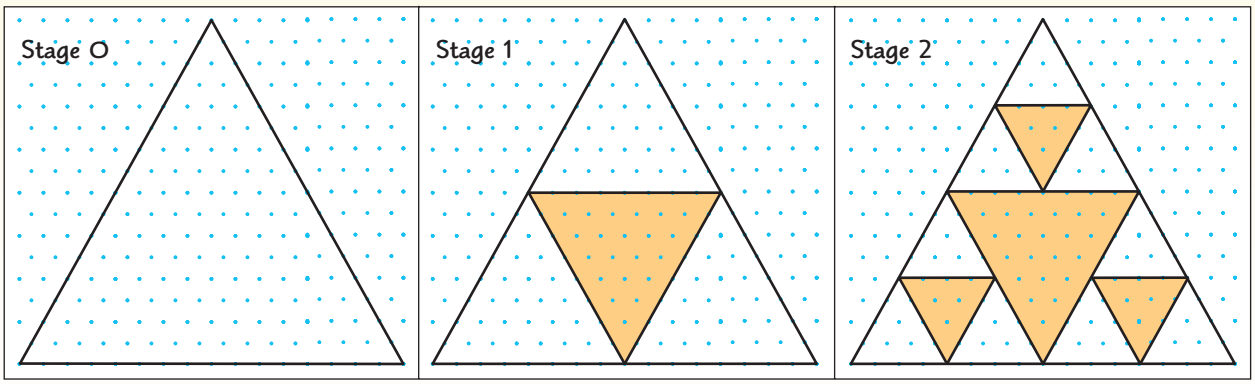
Sierpinski Triangle

Collect Data

Stage 0 On isometric dot paper, draw an equilateral triangle in which each side is 16 units long.

Stage 1 Connect the midpoints of each side to form another triangle. Shade the center triangle.

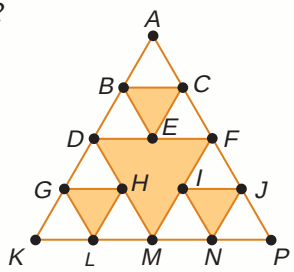
Stage 2 Repeat the process using the three nonshaded triangles. Connect the midpoints of each side to form other triangles.



If you repeat this process indefinitely, the pattern that results is called the **Sierpinski Triangle.** Since this figure is created by repeating the same procedure over and over again, it is an example of a geometric shape called a *fractal*.

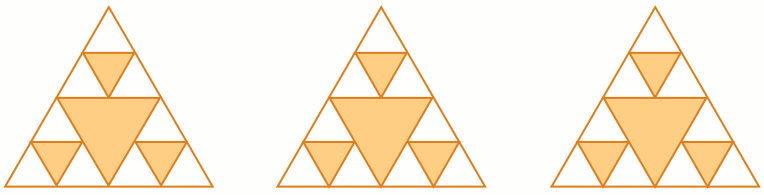
Analyze the Data

1. Continue the process through Stage 4. How many nonshaded triangles do you have at Stage 4?
2. What is the perimeter of a nonshaded triangle in Stage 0 through Stage 4?
3. If you continue the process indefinitely, describe what will happen to the perimeter of each nonshaded triangle.
4. Study $\triangle DFM$ in Stage 2 of the Sierpenski Triangle shown at the right. Is this an equilateral triangle? Are $\triangle BCE$, $\triangle GHL$, or $\triangle IJN$ equilateral?
5. Is $\triangle BCE \sim \triangle DFM$? Explain your answer.
6. How many Stage 1 Sierpinski triangles are there in Stage 2?



Make a Conjecture

7. How can three copies of a Stage 2 triangle be combined to form a Stage 3 triangle?



8. Combine three copies of the Stage 4 Sierpinski triangle. Which stage of the Sierpinski Triangle is this?
9. How many copies of the Stage 4 triangle would you need to make a Stage 6 triangle?

6-6 Fractals and Self-Similarity

What You'll Learn

- Recognize and describe characteristics of fractals.
- Identify nongeometric iteration.

How is mathematics found in nature?

Patterns can be found in many objects in nature, including broccoli. If you take a piece of broccoli off the stalk, the small piece resembles the whole. This pattern of repeated shapes at different scales is part of fractal geometry.

Vocabulary

- iteration
- fractal
- self-similar

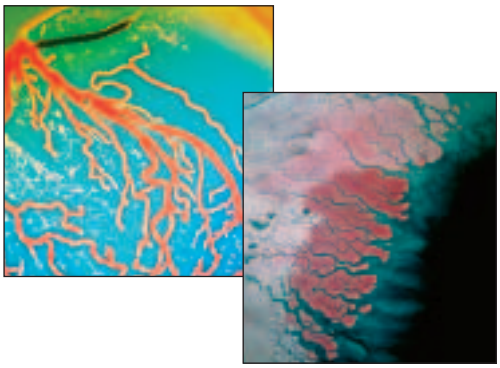
CHARACTERISTICS OF FRACTALS

Benoit Mandelbrot, a mathematician, coined the term *fractal* to describe things in nature that are irregular in shape, such as clouds, coastlines, or the growth of a tree. The patterns found in nature are analyzed and then recreated on a computer, where they can be studied more closely. These patterns are created using a process called **iteration**. Iteration is a process of repeating the same procedure over and over again. A **fractal** is a geometric figure that is created using iteration. The pattern's structure appears to go on infinitely.

WebQuest

By creating a Sierpinski Triangle, you can find a pattern in the area and perimeter of this well-known fractal. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

Compare the pictures of a human circulatory system and the mouth of the Ganges in Bangladesh. Notice how the branches of the tributaries have the same pattern as the branching of the blood vessels.



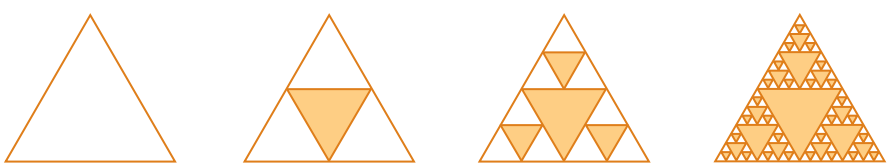
One characteristic of fractals is that they are **self similar**. That is, the smaller and smaller details of a shape have the same geometric characteristics as the original form.

The Sierpinski Triangle is a fractal that is self-similar. Stage 1 is formed by drawing the midsegments of an equilateral triangle and shading in the triangle formed by them. Stage 2 repeats the process in the unshaded triangles. This process can continue indefinitely with each part still being similar to the original.

Study Tip

Sierpinski Triangle

The Sierpenski Triangle can also be shown without shading. The triangles that are used to continue the process are those with a vertex pointing upward.



Stage 0 Stage 1 Stage 2 Stage 4

The Sierpinski Triangle is said to be *strictly self-similar*, which means that any of its parts, no matter where they are located or what size is selected, contain a figure that is similar to the whole.

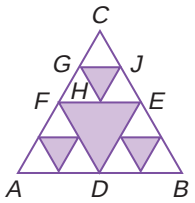
(t)C Squared Studios/PhotoDisc, (bl)CNRI/PhotoTake, (br)CORBIS

CONTENTS

Example 1 Self-Similarity

Prove that a triangle formed in Stage 2 of a Sierpinski triangle is similar to the triangle in Stage 0.

The argument will be the same for any triangle in Stage 2, so we will use only $\triangle CGJ$ from Stage 2.



Given: $\triangle ABC$ is equilateral.
D, E, F, G, J, and H are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$, $\overline{FC}$, $\overline{CE}$, and $\overline{FE}$, respectively.

Prove: $\triangle CGJ \sim \triangle CAB$

Statements	Reasons
1. $\triangle ABC$ is equilateral; D, E, F are midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CA}$; G, J, and H are midpoints of $\overline{FC}$, $\overline{CE}$, $\overline{FE}$.	1. Given
2. $\overline{FE}$ is a midsegment of $\triangle CAB$; $\overline{GJ}$ is a midsegment of $\triangle CFE$.	2. Definition of a Triangle Midsegment
3. $\overline{FE} \parallel \overline{AB}$; $\overline{GJ} \parallel \overline{FE}$	3. Triangle Midsegment Theorem
4. $\overline{GJ} \parallel \overline{AB}$	4. Two segments parallel to the same segment are parallel.
5. $\angle CGJ \cong \angle CAB$	5. Corresponding $\measuredangle$s Postulate
6. $\angle C \cong \angle C$	6. Reflexive Property
7. $\triangle CGJ \sim \triangle CAB$	7. AA Similarity

Thus, using the same reasoning, every triangle in Stage 2 is similar to the original triangle in Stage 0.

Study Tip

Look Back
To review **midsegment**, see Lesson 6-4.

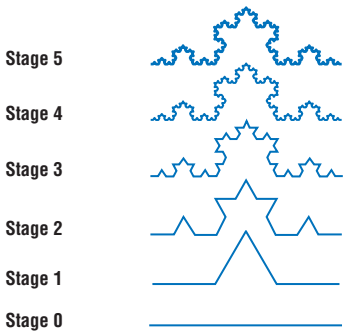
You can generate many other fractal images using an iterative process.

Example 2 Create a Fractal

Draw a segment and trisect it. Create a fractal by replacing the middle third of the segment with two segments of the same length as the removed segment.

After the first geometric iteration, repeat the process on each of the four segments in Stage 1. Continue to repeat the process.

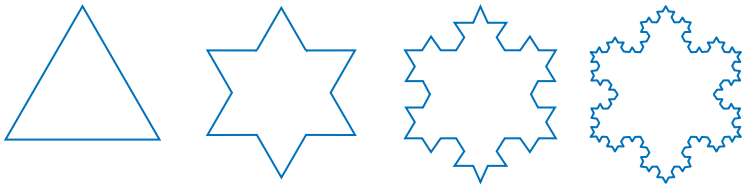
This fractal image is called a *Koch curve*.



Study Tip

Common Misconceptions
Not all repeated patterns are self-similar. The Koch curve is one such example when applied to a triangle.

If the first stage is an equilateral triangle, instead of a segment, this iteration will produce a fractal called *Koch's snowflake*.



CONTENTS

NONGEOMETRIC ITERATION An iterative process does not always include manipulation of geometric shapes. Iterative processes can be translated into formulas or algebraic equations. These are called *recursive formulas*.

Study Tip

Recursion on the Graphing Calculator

To do recursion on a graphing calculator, store 2 as the value for X and press ENTER. Then enter $X^2 : X^2 \rightarrow X$. Press ENTER for each iteration.

Example 3 Evaluate a Recursive Formula

Find the value of x^2, where x initially equals 2. Then use that value as the next x in the expression. Repeat the process four times and describe your observations.

The iterative process is to square the value repeatedly. Begin with $x = 2$. The value of x^2 becomes the next value for x.

x	2	4	16	256	65,536
x^2	4	16	256	65,536	4,294,967,296

The values grow greater with each iteration, approaching infinity.

Example 4 Find a Recursive Formula

PASCAL'S TRIANGLE ***Pascal's Triangle* is a numerical pattern in which each row begins and ends with 1 and all other terms in the row are the sum of the two numbers above it.**

a. Find a formula in terms of the row number for the sum of the values in any row in the Pascal's triangle.

To find the sum of the values in the tenth row, we can investigate a simpler problem. What is the sum of values in the first four rows of the triangle?

Row	Pascal's Triangle	Sum	Pattern
1	1	1	$2^0 = 2^{1-1}$
2	1 1	2	$2^1 = 2^{2-1}$
3	1 2 1	4	$2^2 = 2^{3-1}$
4	1 3 3 1	8	$2^3 = 2^{4-1}$
5	1 4 6 4 1	16	$2^4 = 2^{5-1}$

It appears that the sum of any row is a power of 2. The formula is 2 to a power that is one less than the row number: $A_n = 2^{n-1}$.

b. What is the sum of the values in the tenth row of Pascal's triangle?

The sum of the values in the tenth row will be 2^{10-1} or 512.

Example 5 Solve a Problem Using Iteration

BANKING **Felisa has $2500 in a money market account that earns 3.2% interest. If the interest is compounded annually, find the balance of her account after 3 years.**

First, write an equation to find the balance after one year.

current balance + (current balance × interest rate) = new balance

$$2500 + (2500 \cdot 0.032) = 2580$$

$$2580 + (2580 \cdot 0.032) = 2662.56$$

$$2662.56 + (2662.56 \cdot 0.032) = 2747.76$$

After 3 years, Felisa will have $2747.76 in her account.

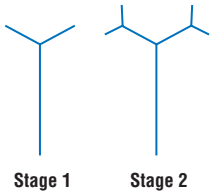
Check for Understanding

Concept Check

1. **Describe** a *fractal* in your own words. Include characteristics of fractals in your answer.
2. **Explain** why computers provide an efficient way to generate fractals.
3. **OPEN ENDED** Find an example of fractal geometry in nature, excluding those mentioned in the lesson.

Guided Practice

For Exercises 4–6, use the following information.
A *fractal tree* can be drawn by making two new branches from the endpoint of each original branch, each one-third as long as the previous branch.



4. Draw Stages 3 and 4 of a fractal tree. How many total branches do you have in Stages 1 through 4? (Do not count the stems.)
5. Find a pattern to predict the number of branches at each stage.
6. Is a fractal tree strictly self-similar? Explain.

For Exercises 7–9, use a calculator.

7. Find the square root of 2. Then find the square root of the result.
8. Find the square root of the result in Exercise 7. What would be the result after 100 repeats of taking the square root?
9. Determine whether this is an iterative process. Explain.

Application

10. **BANKING** Jamir has $4000 in a savings account. The annual percent interest rate is 1.1%. Find the amount of money Jamir will have after the interest is compounded four times.

Practice and Apply

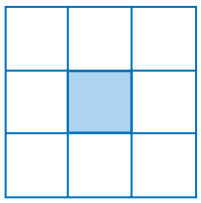
Homework Help

For Exercises	See Examples
11–13, 21–23	1, 2
14–20, 25, 28	4
24–29	2
30–37	3
38–40	5

Extra Practice
See page 766.

For Exercises 11–13, Stage 1 of a fractal is drawn on grid paper so that each side of the large square is 27 units long. The trisection points of the sides are connected to make 9 smaller squares with the middle square shaded. The shaded square is known as a *hole*.

11. Copy Stage 1 on your paper. Then draw Stage 2 by repeating the Stage 1 process in each of the outer eight squares. How many holes are in this stage?



12. Draw Stage 3 by repeating the Stage 1 process in each unshaded square of Stage 2. How many holes are in Stage 3?
13. If you continue the process indefinitely, will the figure you obtain be strictly self-similar? Explain.
14. Count the number of dots in each arrangement. These numbers are called *triangular numbers*. The second triangular number is 3 because there are three dots in the array. How many dots will be in the seventh triangular number?

CONTENTS

For Exercises 15–20, refer to Pascal's triangle on page 327. Look at the third diagonal from either side, starting at the top of the triangle.

15. Describe the pattern.

16. Explain how Pascal's triangle relates to the triangular numbers.

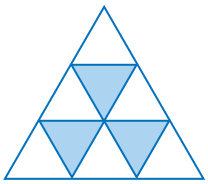
17. Generate eight rows of Pascal's triangle. Replace each of the even numbers with a 0 and each of the odd numbers with a 1. Color each 1 and leave the 0s uncolored. Describe the picture.

18. Generate eight rows of Pascal's triangle. Divide each entry by 3. If the remainder is 1 or 2, shade the number cell black. If the remainder is 0, leave the cell unshaded. Describe the pattern that emerges.

19. Find the sum of the first 25 numbers in the outside diagonal of Pascal's triangle.

20. Find the sum of the first 50 numbers in the second diagonal.

The three shaded interior triangles shown were made by trisecting the three sides of an equilateral triangle and connecting the points.



21. Prove that one of the nonshaded triangles is similar to the original triangle.

22. Repeat the iteration once more.

23. Is the new figure strictly self-similar?

24. How many nonshaded triangles are in Stages 1 and 2?

Refer to the Koch Curve on page 326.

25. What is a formula for the number of segments in terms of the stage number? Use your formula to predict the number of segments in Stage 8 of a Koch curve.

26. If the length of the original segment is 1 unit, how long will the segments be in each of the first four stages? What will happen to the length of each segment as the number of stages continues to increase?

Refer to the Koch Snowflake on page 326. At Stage 1, the length of each side is 1 unit.

27. What is the perimeter at each of the first four stages of a Koch snowflake?

28. What is a formula for the perimeter in terms of the stage number? Describe the perimeter as the number of stages continues to increase.

29. Write a paragraph proof to show that the triangles generated on the sides of a Koch Snowflake in Stage 1, are similar to the original triangle.

Find the value of each expression. Then use that value as the next x in the expression. Repeat the process four times, and describe your observations.

30. $\sqrt{x}$, where x initially equals 12

31. $\frac{1}{x}$, where x initially equals 5

32. $x^{\frac{1}{3}}$, where x initially equals 0.3

33. 2^x, where x initially equals 0

Find the first three iterates of each expression.

34. $2x + 1$, x initially equals 1

35. $x - 5$, where x initially equals 5

36. $x^2 - 1$, x initially equals 2

37. $3(2 - x)$, where x initially equals 4

38. BANKING Raini has a credit card balance of \$1250 and a monthly interest rate of 1.5%. If he makes payments of \$100 each month, what will the balance be after 3 months?

More About . . .

Blaise Pascal

Pascal (1623–1662) did not discover Pascal's triangle, but it was named after him in honor of his work in 1653 called *Treatise on the Arithmetical Triangle*. The patterns in the triangle are also used in probability.

Source: *Great Moments in Mathematics/After 1650*

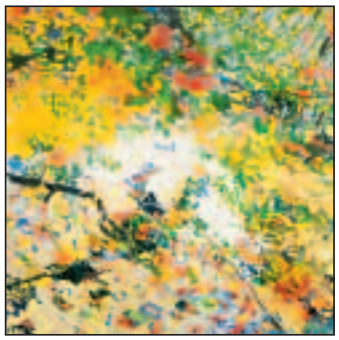
Reunion des Musees Nationaux/Art Resource, NY

WEATHER **For Exercises 39 and 40, use the following information.**
There are so many factors that affect the weather that it is difficult for meteorologists to make accurate long term predictions. Edward N. Lorenz called this dependence *the Butterfly Effect* and posed the question "Can the flap of a butterfly's wings in Brazil cause a tornado in Texas?"

39. Use a calculator to find the first ten iterates of $4x(1 - x)$ when x initially equals 0.200 and when the initial value is 0.201. Did the change in initial value affect the tenth value?

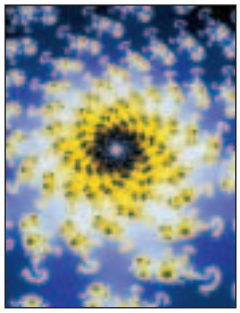
40. Why do you think this is called the Butterfly Effect?

41. **ART** Describe how artist Jean-Paul Agosti used iteration and self-similarity in his painting *Jardin du Creuset*.



42. **NATURE** Some of these pictures are of real objects and others are fractal images of objects.

a. Compare the pictures and identify those you think are of real objects.
b. Describe the characteristics of fractals shown in the images.


flower

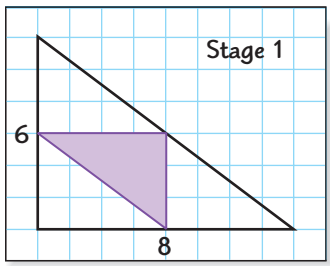
mountain

feathers

moss

43. **RESEARCH** Use the Internet or other sources to find the names and pictures of the other fractals Waclaw Sierpinski developed.

44. **CRITICAL THINKING** Draw a right triangle on grid paper with 6 and 8 units for the lengths of the perpendicular sides. Shade the triangle formed by the three midsegments. Repeat the process for each unshaded triangle. Find the perimeter of the shaded triangle in Stage 1. What is the total perimeter of all the shaded triangles in Stage 2?



45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is mathematics related to nature?

Include the following in your answer:

- explain why broccoli is an example of fractal geometry, and
- how scientists can use fractal geometry to better understand nature.

(t)Courtesy Jean-Paul Agosti, (bl)Stephen Johnson/Getty Images, (bcl)Gregory Sams/Science Photo Library/Photo Researchers, (br)Gail Meese
CONTENTS

Standardized Test Practice

46. GRID IN A triangle has side lengths of 3 inches, 6 inches, and 8 inches. A similar triangle is 24 inches on one side. Find the maximum perimeter, in inches, of the second triangle.

47. ALGEBRA A repair technician charges \$80 for the first thirty minutes of each house call plus \$2 for each additional minute. The repair technician charged a total of \$170 for a job. How many minutes did the repair technician work?

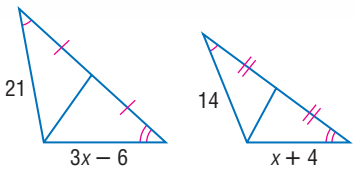
(A) 45 min (B) 55 min (C) 75 min (D) 85 min

Maintain Your Skills

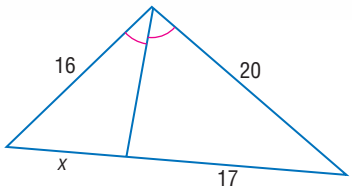
Mixed Review

Find x. *(Lesson 6-5)*

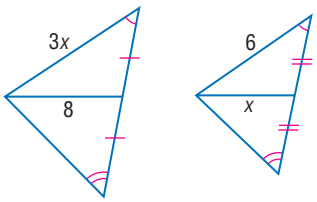
48.



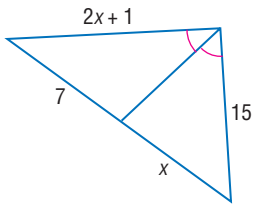
49.



50.



51.

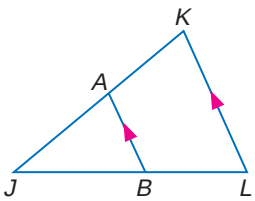


For Exercises 52–54, refer to $\triangle JKL$. *(Lesson 6-4)*

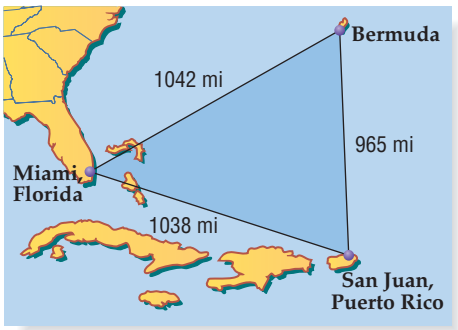
52. If $JL = 27$, $BL = 9$, and $JK = 18$, find JA.

53. If $AB = 8$, $KL = 10$, and $JB = 13$, find JL.

54. If $JA = 25$, $AK = 10$, and $BL = 14$, find JB.

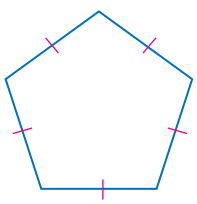


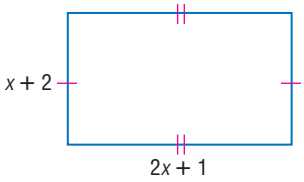
55. FOLKLORE The Bermuda Triangle is an imaginary region located off the southeastern Atlantic coast of the United States. It is the subject of many stories about unexplained losses of ships, small boats, and aircraft. Use the vertex locations to name the angles in order from least measure to greatest measure. *(Lesson 5-4)*

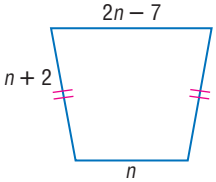


Find the length of each side of the polygon for the given perimeter. *(Lesson 1-6)*

56. $P = 60$ centimeters **57.** $P = 54$ feet **58.** $P = 57$ units







Chapter 6

Study Guide and Review

Vocabulary and Concept Check

cross products (p. 283)
extremes (p. 283)
fractal (p. 325)
iteration (p. 325)
means (p. 283)
midsegment (p. 308)
proportion (p. 283)
ratio (p. 282)
scale factor (p. 290)
self-similar (p. 325)
similar polygons (p. 289)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **State whether each sentence is *true* or *false*. If false, replace the underlined expression to make a true sentence.**

1. A midsegment of a triangle is a segment whose endpoints are the midpoints of two sides of the triangle.
2. Two polygons are similar if and only if their corresponding angles are congruent and the measures of the corresponding sides are congruent.
3. If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.
4. If two triangles are similar, then the perimeters are proportional to the measures of the corresponding angles.
5. A fractal is a geometric figure that is created using *recursive formulas*.
6. A midsegment of a triangle is parallel to one side of the triangle, and its length is twice the length of that side.
7. For any numbers a and c and any nonzero numbers b and d, $\frac{a}{b} = \frac{c}{d}$ if and only if $ad = bc$.
8. If two triangles are similar, then the measures of the corresponding angle bisectors of the triangle are proportional to the measures of the corresponding sides.
9. If a line intersects two sides of a triangle and separates the sides into corresponding segments of proportional lengths, then the line is equal to one-half the length of the third side.

Lesson-by-Lesson Review

6-1 Proportions

See pages 282–287.

Concept Summary

- A ratio is a comparison of two quantities.
- A proportion is an equation stating that two ratios are equal.

Example Solve $\frac{z}{40} = \frac{5}{8}$.

$\frac{z}{40} = \frac{5}{8}$	Original proportion
$z \cdot 8 = 40(5)$	Cross products
$8z = 200$	Multiply.
$z = 25$	Divide each side by 8.

www.geometryonline.com/vocabulary_review

CONTENTS

Exercises **Solve each proportion.** *See Example 3 on page 284.*

10. $\frac{3}{4} = \frac{x}{12}$ **11.** $\frac{7}{3} = \frac{28}{z}$ **12.** $\frac{x+2}{5} = \frac{14}{10}$

13. $\frac{3}{7} = \frac{7}{y-3}$ **14.** $\frac{4-x}{3+x} = \frac{16}{25}$ **15.** $\frac{x-12}{6} = \frac{x+7}{-4}$

16. BASEBALL A player's slugging percentage is the ratio of the number of total bases from hits to the number of total at-bats. The ratio is converted to a decimal (rounded to three places) by dividing. If Alex Rodriguez of the Texas Rangers has 263 total bases in 416 at-bats, what is his slugging percentage?

17. A 108-inch-long board is cut into two pieces that have lengths in the ratio 2:7. How long is each new piece?

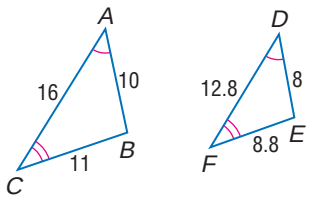
6-2 Similar Polygons

See pages 289–297.

Concept Summary

- In similar polygons, corresponding angles are congruent, and corresponding sides are in proportion.
- The ratio of two corresponding sides in two similar polygons is the scale factor.

Example **Determine whether the pair of triangles is similar. Justify your answer.**



$\angle A \cong \angle D$ and $\angle C \cong \angle F$, so by the Third Angle Theorem, $\angle B \cong \angle E$. All of the corresponding angles are congruent.

Now, check to see if corresponding sides are in proportion.

$\frac{AB}{DE} = \frac{10}{8}$ $= \frac{5}{4}$ or 1.25

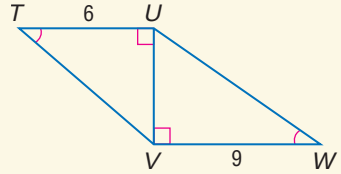
$\frac{BC}{EF} = \frac{11}{8.8}$ $= \frac{5}{4}$ or 1.25

$\frac{CA}{FD} = \frac{16}{12.8}$ $= \frac{5}{4}$ or 1.25

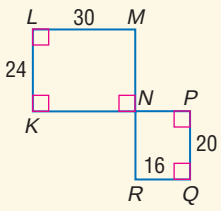
The corresponding angles are congruent, and the ratios of the measures of the corresponding sides are equal, so $\triangle ABC \sim \triangle DEF$.

Exercises **Determine whether each pair of figures is similar. Justify your answer.**
See Example 1 on page 290.

18.



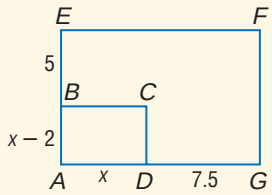
19.



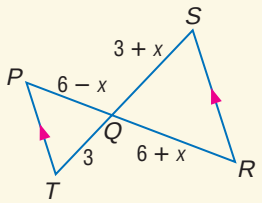
CONTENTS

Each pair of polygons is similar. Write a similarity statement, and find x, the measures of the indicated sides, and the scale factor. *See Example 3 on page 291.*

20. $\overline{AB}$ and $\overline{AG}$



21. $\overline{PQ}$ and $\overline{QS}$



6-3 Similar Triangles

See pages 298–306.

Concept Summary

- AA, SSS, and SAS Similarity can all be used to prove triangles similar.
- Similarity of triangles is reflexive, symmetric, and transitive.

Example

INDIRECT MEASUREMENT **Alonso wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 P.M., he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree?**

Since the triangles formed are similar, a proportion can be written. Let x be the height of the tree.

$$\frac{\text{height of the tree}}{\text{height of the fence}} = \frac{\text{tree shadow length}}{\text{fence shadow length}}$$

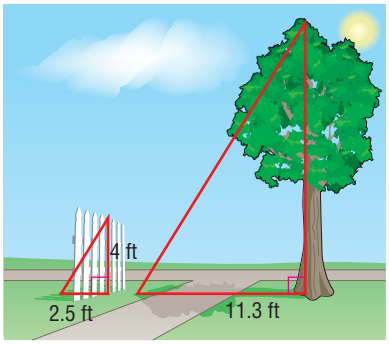
$$\frac{x}{4} = \frac{11.3}{2.5} \quad \text{Substitution}$$

$$x \cdot 2.5 = 4(11.3) \quad \text{Cross products}$$

$$2.5x = 45.2 \quad \text{Simplify.}$$

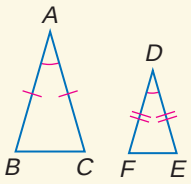
$$x = 18.08 \quad \text{Divide each side by 2.5.}$$

The height of the tree is 18.08 feet.

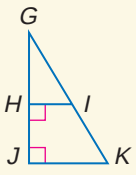


Exercises **Determine whether each pair of triangles is similar. Justify your answer.** *See Example 1 on page 299.*

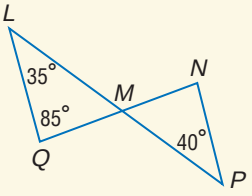
22.



23.

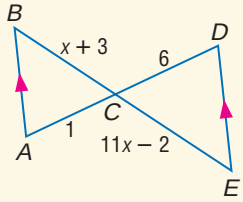


24.

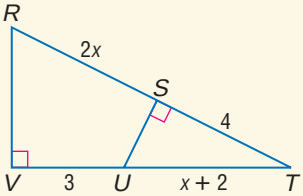


Identify the similar triangles. Find x. *See Example 2 on page 300.*

25.



26.



CONTENTS

6-4 Parallel Lines and Proportional Parts

See pages 307–315.

Concept Summary

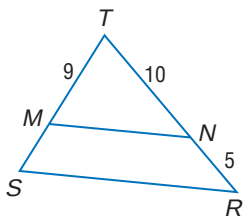
- A segment that intersects two sides of a triangle and is parallel to the third side divides the two intersected sides in proportion.
- If two lines divide two segments in proportion, then the lines are parallel.

Example **In $\triangle TRS$, $TS = 12$. Determine whether $\overline{MN} \parallel \overline{SR}$.**

If $TS = 12$, then $MS = 12 - 9$ or 3. Compare the segment lengths to determine if the lines are parallel.

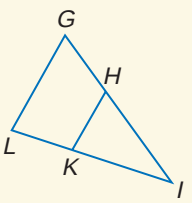
$\frac{TM}{MS} = \frac{9}{3} = 3 \qquad \frac{TN}{NR} = \frac{10}{5} = 2$

Because $\frac{TM}{MS} \neq \frac{TN}{NR}$, $\overline{MN} \nparallel \overline{SR}$.



Exercises **Determine whether $\overline{GL} \parallel \overline{HK}$. Justify your answer.**

See Example 2 on page 308.



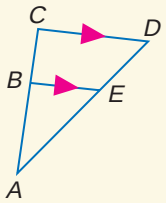
27. $IH = 21$, $HG = 14$, $LK = 9$, $KI = 15$

28. $GH = 10$, $HI = 35$, $IK = 28$, $IL = 36$

29. $GH = 11$, $HI = 22$, and IL is three times the length of $\overline{KL}$.

30. $LK = 6$, $KI = 18$, and IG is three times the length of $\overline{HI}$.

Refer to the figure at the right. *See Example 1 on page 308.*



31. Find ED if $AB = 6$, $BC = 4$, and $AE = 9$.

32. Find AE if $AB = 12$, $AC = 16$, and $ED = 5$.

33. Find CD if $AE = 8$, $ED = 4$, and $BE = 6$.

34. Find BC if $BE = 24$, $CD = 32$, and $AB = 33$.

6-5 Parts of Similar Triangles

See pages 316–323.

Concept Summary

- Similar triangles have perimeters proportional to the corresponding sides.
- Corresponding angle bisectors, medians, and altitudes of similar triangles have lengths in the same ratio as corresponding sides.

Example **If $\overline{FB} \parallel \overline{EC}$, $\overline{AD}$ is an angle bisector of $\angle A$, $BF = 6$, $CE = 10$, and $AD = 5$, find AM.**

By AA Similarity using $\angle AFE \cong \angle ABF$ and $\angle A \cong \angle A$, $\triangle ABF \sim \triangle ACE$.

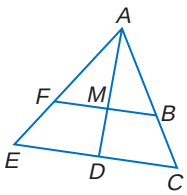
$\frac{AM}{AD} = \frac{BF}{CE}$ $\sim\triangle$s have angle bisectors in the same proportion as the corresponding sides.

$\frac{x}{5} = \frac{6}{10}$ $AD = 5$, $AF = 6$, $FE = 4$, $AM = x$

$10x = 30$ Cross products

$x = 3$ Divide each side by 10.

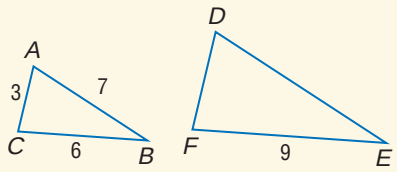
Thus, $AM = 3$.



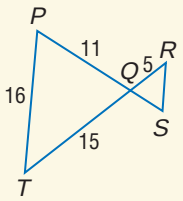
CONTENTS

Exercises **Find the perimeter of the given triangle.** *See Example 1 on page 316.*

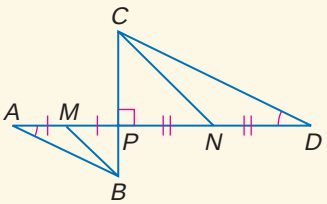
35. $\triangle DEF$ if $\triangle DEF \sim \triangle ABC$



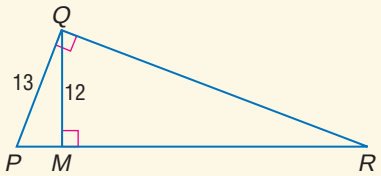
36. $\triangle QRS$ if $\triangle QRS \sim \triangle QTP$



37. $\triangle CPD$ if the perimeter of $\triangle BPA$ is 12, $BM = \sqrt{13}$, and $CN = 3\sqrt{13}$



38. $\triangle PQR$, if $\triangle PQM \sim \triangle PRQ$



6-6 Fractals and Self-Similarity

See pages 325–331.

Concept Summary

- Iteration is the creation of a sequence by repetition of the same operation.
- A fractal is a geometric figure created by iteration.
- An iterative process involving algebraic equations is a recursive formula.

Example

Find the value of $\frac{x}{2} + 4$, where x initially equals -8. Use that value as the next x in the expression. Repeat the process five times and describe your observations.

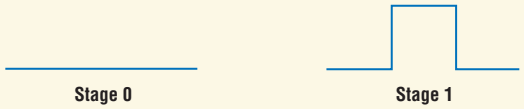
Make a table to organize each iteration.

Iteration	1	2	3	4	5	6
x	−8	0	4	6	7	7.5
$\frac{x}{2}+4$	0	4	6	7	7.5	7.75

The x values appear to get closer to the number 8 with each iteration.

Exercises **Draw Stage 2 of the fractal shown below. Determine whether Stage 2 is similar to Stage 1.** *See Example 2 on page 326.*

39.



Find the first three iterates of each expression. *See Example 3 on page 327.*

40. $x^3 - 4$, x initially equals 2

41. $3x + 4$, x initially equals -4

42. $\frac{1}{x}$, x initially equals 10

43. $\frac{x}{10} - 9$, x initially equals 30

Chapter 6

Practice Test

Vocabulary and Concepts

Choose the answer that best matches each phrase.

1. an equation stating that two ratios are equal
2. the ratio between corresponding sides of two similar figures
3. the means multiplied together and the extremes multiplied together

a. scale factor
b. proportion
c. cross products

Skills and Applications

Solve each proportion.

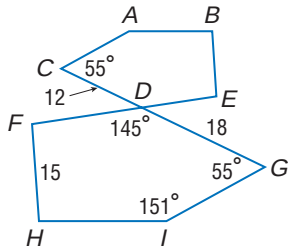
4. $\frac{x}{14} = \frac{1}{2}$

5. $\frac{4x}{3} = \frac{108}{x}$

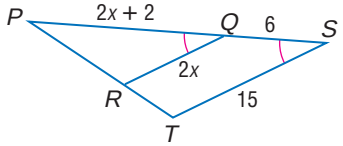
6. $\frac{k+2}{7} = \frac{k-2}{3}$

Each pair of polygons is similar. Write a similarity statement and find the scale factor.

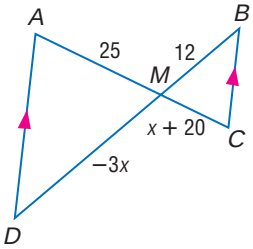
7.



8.

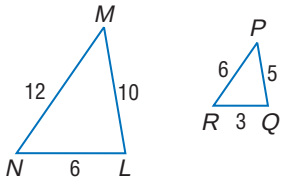


9.

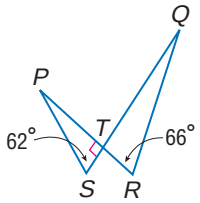


Determine whether each pair of triangles is similar. Justify your answer.

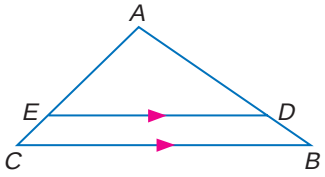
10.



11.

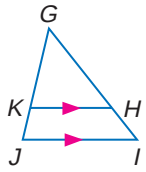


12.



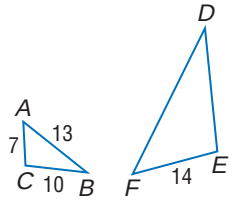
Refer to the figure at the right.

13. Find KJ if $GJ = 8$, $GH = 12$, and $HI = 4$.
14. Find GK if $GI = 14$, $GH = 7$, and $KJ = 6$.
15. Find GI if $GH = 9$, $GK = 6$, and $KJ = 4$.

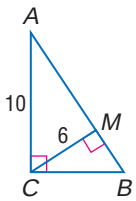


Find the perimeter of the given triangle.

16. $\triangle DEF$, if $\triangle DEF \sim \triangle ACB$



17. $\triangle ABC$



18. Find the first three iterates of $5x + 27$ when x initially equals -3.

19. **BASKETBALL** Terry wants to measure the height of the top of the backboard of his basketball hoop. At 4:00, the shadow of a 4-foot fence is 20 inches, and the shadow of the backboard is 65 inches. What is the height of the top of the backboard?

20. **STANDARDIZED TEST PRACTICE** If a person's weekly salary is $\$X$ and $\$Y$ is saved, what part of the weekly salary is spent?

(A) $\frac{X}{Y}$ (B) $\frac{X-Y}{X}$ (C) $\frac{X-Y}{Y}$ (D) $\frac{Y-X}{Y}$

Chapter 6 Standardized Test Practice

Part 1 Multiple Choice

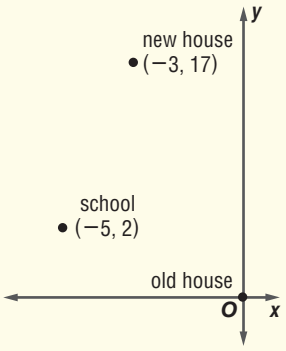
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Which of the following is equivalent to $|-8 + 2|$? (Prerequisite Skill)

Ⓐ 10 Ⓑ 6 Ⓒ −6 Ⓓ −10

2. Kip's family moved to a new house. He used a coordinate plane with units in miles to locate his new house and school in relation to his old house. What is the distance between his new house and school? (Lesson 1-3)

Ⓐ 12 miles
Ⓑ $\sqrt{229}$ miles
Ⓒ 17 miles
Ⓓ $\sqrt{425}$ miles



3. The diagonals of rectangle $ABCD$ are $\overline{AC}$ and $\overline{BD}$. Hallie found that the distances from the point where the diagonals intersect to each vertex were the same. Which of the following conjectures could Hallie make? (Lesson 2-1)

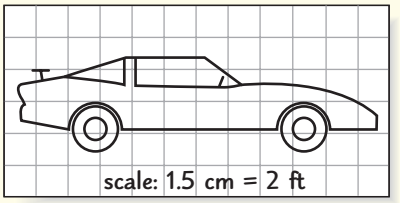
Ⓐ Diagonals of a rectangle are congruent.
Ⓑ Diagonals of a rectangle create equilateral triangles.
Ⓒ Diagonals of a rectangle intersect at more than one point.
Ⓓ Diagonals of a rectangle are congruent to the width.

4. If two sides of a triangular sail are congruent, which of the following terms *cannot* be used to describe the shape of the sail? (Lesson 4-1)

Ⓐ acute Ⓑ equilateral
Ⓒ obtuse Ⓓ scalene

5. Miguel is using centimeter grid paper to make a scale drawing of his favorite car. Miguel's drawing is 11.25 centimeters wide. How many feet long is the actual car? (Lesson 6-1)

Ⓐ 15.0 ft
Ⓑ 18.75 ft
Ⓒ 22.5 ft
Ⓓ 33.0 ft

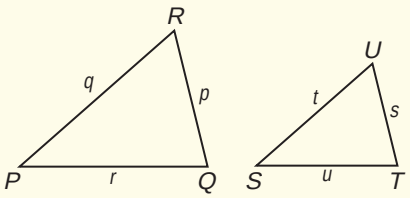


6. Joely builds a large corkboard for her room that is 45 inches tall and 63 inches wide. She wants to build a smaller corkboard with a similar shape for the kitchen. Which of the following could be the dimensions of that corkboard? (Lesson 6-2)

Ⓐ 4 in. by 3 in.
Ⓑ 7 in. by 5 in.
Ⓒ 12 in. by 5 in.
Ⓓ 21 in. by 14 in.

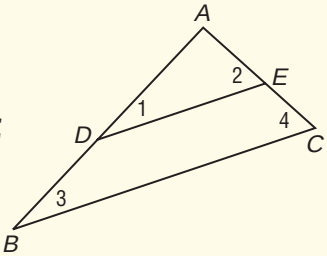
7. If $\triangle PQR$ and $\triangle STU$ are similar, which of the following is a correct proportion? (Lesson 6-3)

Ⓐ $\frac{s}{u} = \frac{t}{q}$
Ⓑ $\frac{s}{u} = \frac{p}{q}$
Ⓒ $\frac{s}{u} = \frac{p}{r}$
Ⓓ $\frac{s}{u} = \frac{r}{p}$



8. In $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$. Which of the following is *not* true? (Lesson 6-4)

Ⓐ $\frac{AD}{DB} = \frac{AE}{EC}$
Ⓑ $\overline{DE} \parallel \overline{BC}$
Ⓒ $\triangle ABC \sim \triangle ADE$
Ⓓ $\angle 1 \cong \angle 4$



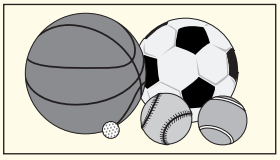
CONTENTS

Part 2 Short Response/Grid In

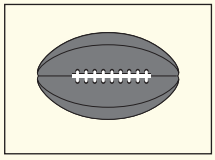
Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

9. During his presentation, Dante showed a picture of several types of balls used in sports. From this picture, he conjectured that all balls used in sports are spheres. Brianna then showed another ball. What is this type of example called? (Lesson 2-1)

Dante

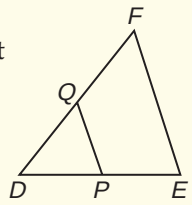


Brianna

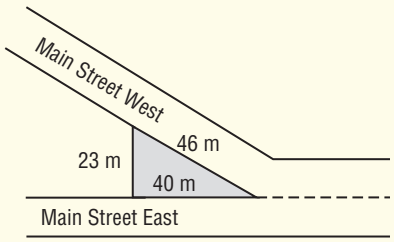


10. What is the equation of a line with slope 3 that contains $A(2, 2)$? (Lesson 3-4)

11. In $\triangle DEF$, P is the midpoint of $\overline{DE}$, and Q is the midpoint of side $\overline{DF}$. If $EF = 3x + 4$ and $PQ = 20$, what is the value of x? (Lesson 6-4)



12. A city planner designs a triangular traffic median on Main Street to provide more green space in the downtown area. The planner builds a model so that the section of the median facing Main Street East measures 20 centimeters. What is the perimeter, in centimeters, of the model of the traffic median? (Lesson 6-5)



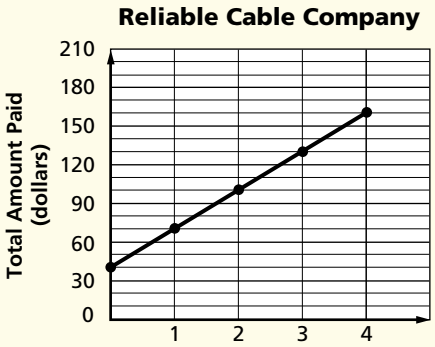
The Princeton Review Test-Taking Tip

Question 7 In similar triangles, corresponding angles are congruent and corresponding sides are proportional. When you set up a proportion, be sure that it compares corresponding sides. In Question 7, p corresponds to s, q corresponds to t, and r corresponds to u.

Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

13. A cable company charges a one-time connection fee plus a monthly flat rate as shown in the graph.



a. What is the slope of the line that joins the points on the graph? (Lesson 3-3)

b. Discuss what the value of the slope represents. (Lesson 3-3)

c. Write the equation of the line. (Lesson 3-4)

d. If the company presents a special offer that lowers the monthly rate by \$5, how will the equation and graph change? (Lesson 3-4)

14. In $\triangle ADE$, $\overline{BC}$ is equidistant from $\overline{DE}$.

a. Prove that $\triangle ABC \sim \triangle ADE$. (Lessons 6-3 and 6-4)

b. Suppose $AB = 3500$ feet, $BD = 1500$ feet, and $BC = 1400$ feet. Find DE. (Lesson 6-3)